2.12 Introduction to Robotics
Problem Set No.4
Out: October 12, 2005 Due: October 19, 2005

Problem 1

An astronaut is operating a shuttle manipulator with an inspection end effecter attached to the tip of the arm. For the sake of simplicity we consider only the three revolute joints, $\theta_1, \theta_2, \theta_3$, and the three links, as shown in the figure. A Cartesian coordinate system, *O-xy*, is attached to the object to be inspected. The distance of the coordinate origin O is L from the location of the first joint. Answer the following questions using the notation shown in the figure.

a) Obtain the forward kinematic equations relating the end effecter position and orientation, x_e, y_e, φ_e, to the three joint angles, $\theta_1, \theta_2, \theta_3$. Note that the end effecter position and orientation are viewed from the Cartesian coordinate system attached to the object, *O-xy*.

b) Obtain the Jacobian matrix associated with the kinematic equations of Part a). Sketch a block diagram of Resolved Motion Rate Control where the astronaut uses a joystick for generating velocity commands, $v_x = \dot{x}_e, v_y = \dot{y}_e, \omega = \dot{\varphi}_e$, with reference to the Cartesian coordinate system *O-xy*.

c) The inspection end effecter must be moved along the object surface, i.e. the *x*-axis, at a constant speed, $v_{xd} = 20 \ cm/\sec$. The gap between the inspection sensor and the object surface must be constant, $y_{ed} = 10 \ cm$, and the orientation must be kept horizontal, $\varphi_{ed} = 90^o$. Compute the time trajectories of the three joint angles as well as the joint velocities when the end effecter moves from Point A at $x = 0, \ y = 10 \ cm$, to Point B at $x = 15 \ m, \ y = 10 \ cm$. The link lengths are $\ell_1 = 10 \ m, \ \ell_2 = 10 \ m, \ \ell_3 = 40 \ cm$, and the distance to the object is $L = 10.5 \ m$. Plot position and velocity profiles using MATLAB.
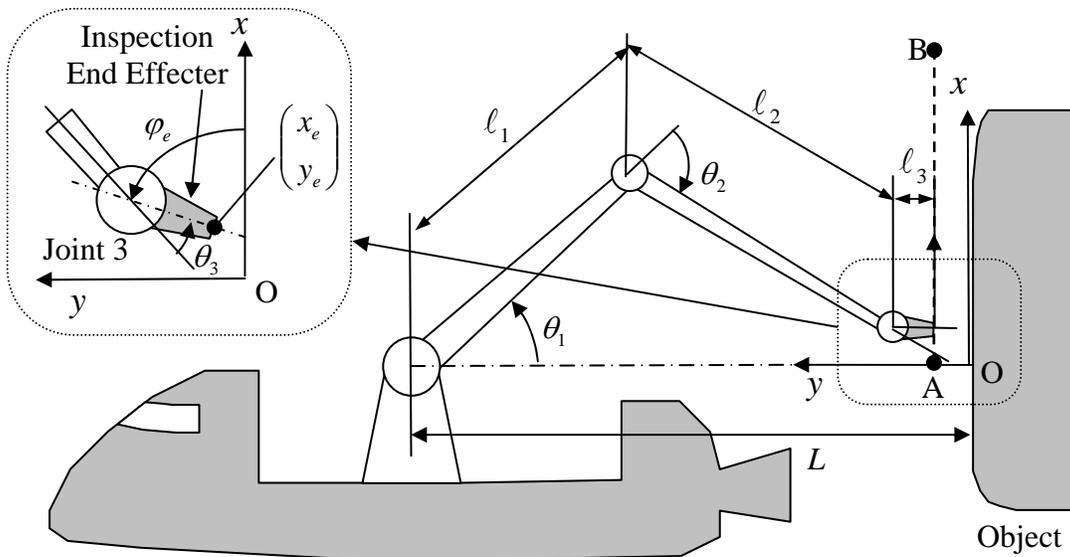

Figure 1 Shuttle manipulator inspecting an object surface
All the angles are measured in the right hand sense. θ_2 in the figure is therefore negative.

Problem 2

Shown below is the same articulated robot as the one in the previous problem set. The robot has three revolute joints that allow the endpoint to move in three-dimensional space. However, this robot has some singular points inside the workspace. Answer the following questions.

a) Obtain each column vector of the Jacobian matrix based on its geometric interpretation, as discussed in class. (Consider the endpoint velocity created by each of the joints while immobilizing the other joints.)

b) Obtain the Jacobian via direct differentiation of the kinematic equations relating the end-effecter coordinates to joint displacements. Compare the result with the geometric approach in part a).

c) For this question and the next question only, assume $\ell_1 = \ell_2 = 1$ for brevity. Compute the Jacobian matrix for the arm configuration:

$$\theta_1 = \frac{\pi}{2}, \ \theta_2 = \frac{\pi}{6}, \ \theta_3 = -\frac{2\pi}{3}$$

and obtain the determinant of the Jacobian.

d) Obtain the joint velocities that move the endpoint with desired velocities $v_x = 1, \ v_y = 2, \ v_z = 0$ at the instant of the arm configuration in part c).

e) Obtain the joint angles of singular configurations by solving the singularity condition: $\det \mathbf{J} = 0$.

f) Based on the results of part e), sketch the arm posture for each of the singular configurations. Show where in the workspace it becomes singular and in which direction the endpoint cannot be moved at a non-zero velocity.
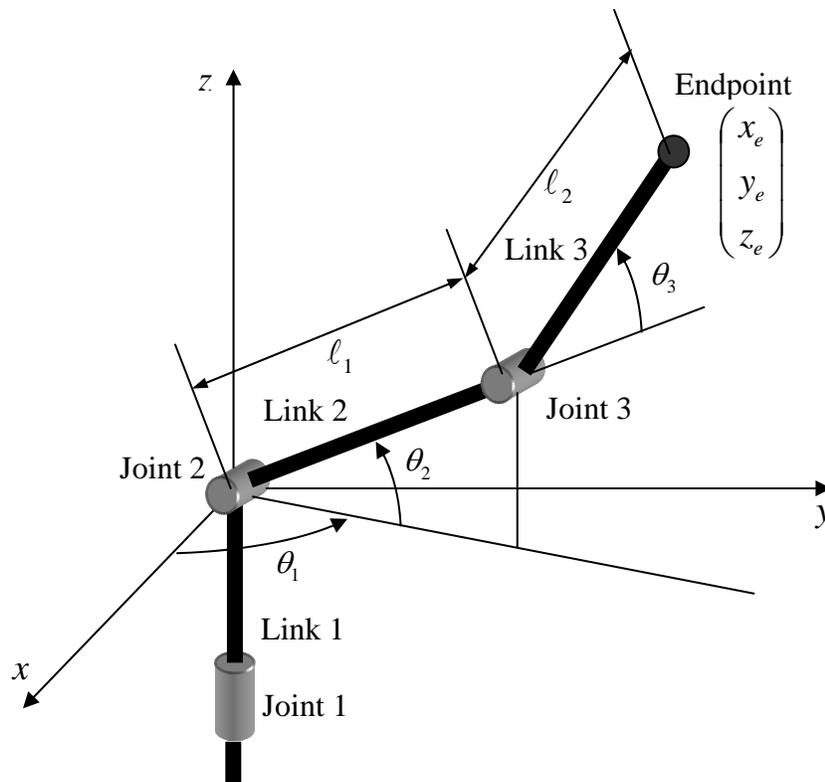

Figure 2 Schematic of a three dof articulated robot

Problem 3

Shown below is a planar 3 d.o.f. robotic leg standing on the ground. Three joint angles, $\theta_1, \theta_2, \theta_3$, all measured from the ground, are used as an independent set of generalized coordinates uniquely locating the system. The second figure below shows the front view of the robot including actuators and transmission mechanisms. Actuator 1 generates torque τ_1 between link 0 and link 1. Note that the body of Actuator 1 is fixed to link 0, while its output shaft is connected to link 1. Actuator 2 is fixed to Link 3, and its output torque τ_2 is transmitted to Joint 2, i.e. the knee joint, through the mass-less belt-pulley system with a gear ratio of 1:1. Actuator 3 is fixed to Link 3, while its output shaft is connected to Link 2. All actuator torques τ_1, τ_2, τ_3 are measured in a right hand sense, as shown by the arrows in the figure. Displacements of the individual actuators are denoted ϕ_1, ϕ_2, ϕ_3, and are measured in the same direction of the torque. The location of the hip, i.e. Link 3, is represented by the coordinates of its center of mass, x_h, y_h, and angle α measured from the base coordinate system fixed to Joint 1, as shown in the figure.

a). Obtain the Jacobian matrix relating infinitesimal joint angles $\theta_1, \theta_2, \theta_3$ to infinitesimal changes to the hip position and orientation, x_h, y_h, α.

b). Obtain the Jacobian matrix relating joint velocities $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$ to actuator angular velocities $\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3$.

c). Obtain actuator angular velocities $\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3$ when the hip is moving horizontally at a constant speed, $\dot{x}_h = V$, $\dot{y}_h = 0$, $\dot{\alpha} = 0$.
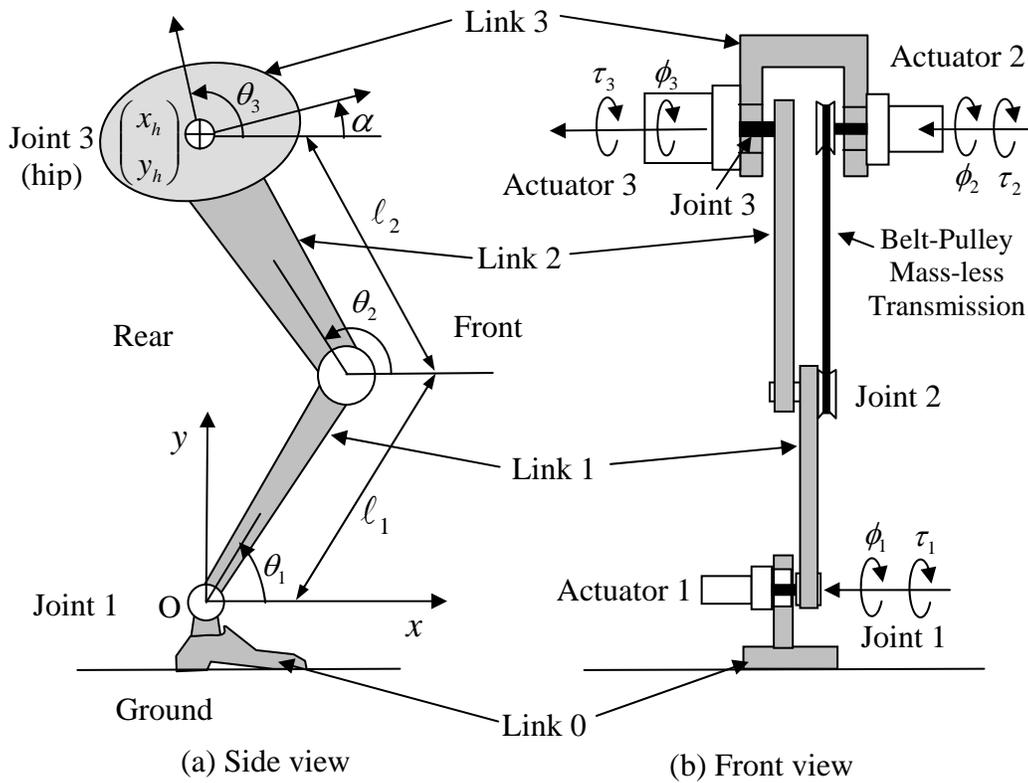

(a) Side view   (b) Front view

Figure 3 Leg robot