2.092/2.093

FINITE ELEMENT OF SOLIDS AND FLUIDS I

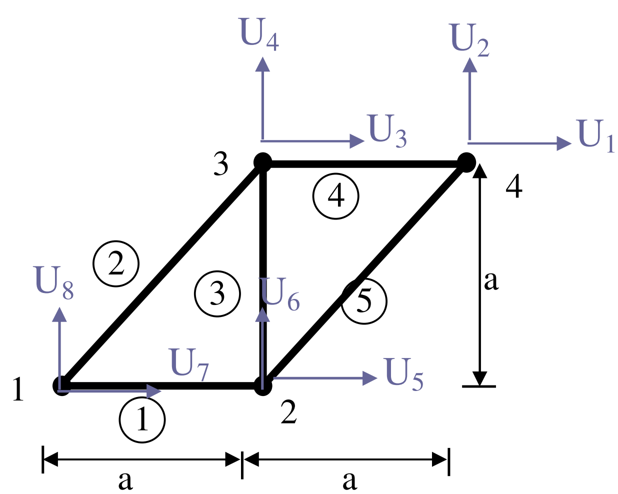
FALL 2009

Homework 1- solution

Instructor:	Prof. K. J. Bathe	Assigned:	Session 3
TA:	Seounghyun Ham	Due:	Session 5

Problem 1 (30 points):

The truss geometry is shown below. Bar mumbers are circled. Joint numbers are placed adjacent to their respective joints.



For a linear static analysis, we have:

$$K_{8\times8}\mathbf{U}_{8\times1}=\mathbf{R}_{8\times1}$$

a) The **K** matrix is calculated column by column. The i[th] column of the stiffness matrix represents the external force vector required to give the structure unit displacement about the i[th] degree of freedom and zero displacement about all other degree of freedom. Take a look at one example how to construct it.

Calculate column 5:

Imposing the following displacement pattern:

$$U_5=1,\ U_1=U_2=U_3=U_4=U_6=U_7=U_8=0$$

The resulting external force vector under this set of displacement conditions is equal to the 5th column of the stiffness matrix **K**. In this case, the truss bar 1 changes length by 1, the truss bar 5 shrinks by $1/\sqrt{2}$, and all other truss bars are fixed in length. Hence, the bar axial forces are as follows.

$$N_1 = \frac{AE}{a}, N_2 = N_3 = N_4 = 0, N_5 = -\frac{AE}{2a}$$

Positive and negative signs of the axial forces imply tension and compression, respectively. Hence the reaction forces, the entries of the 5th column, are obtained from the equilibrium equations at the joints.

$$K_{15} = K_{25} = -\frac{AE}{2\sqrt{2}a}, K_{55} = \frac{AE}{a}(\frac{1}{2\sqrt{2}}+1), K_{65} = \frac{AE}{2\sqrt{2}a}, K_{75} = -\frac{AE}{a}, K_{35} = K_{45} = K_{85} = 0$$

After assembling all columns, the following **K** is determined:

$$\mathbf{K} = \frac{\mathrm{EA}}{\mathrm{a}}\begin{bmatrix} \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} & -1 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 \\ -1 & 0 & \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}}+1 & 0 & -1 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} & -1 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & -1 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}}+1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -1 & 0 & \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} \\ 0 & 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{bmatrix}$$

b) Since $U_1 = U_2 = U_4 = U_7 = U_8 = 0$, we can reduce $K_{8\times8}\mathbf{U}_{8\times1}\mathbf{=}\mathbf{R}_{8\times1}$ to $K_{aa}\mathbf{U}_a\mathbf{=}\mathbf{R}_a$ as follows:

$$\frac{\text{EA}}{\text{a}}\begin{bmatrix} \frac{1}{2\sqrt{2}}+1 & 0 & 0 \\ 0 & \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} \\ 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}}+1 \end{bmatrix}\begin{bmatrix} U_3 \\ U_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} R_3 \\ R_5 \\ R_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 6\times10^4 \\ 0 \end{bmatrix} N.$$

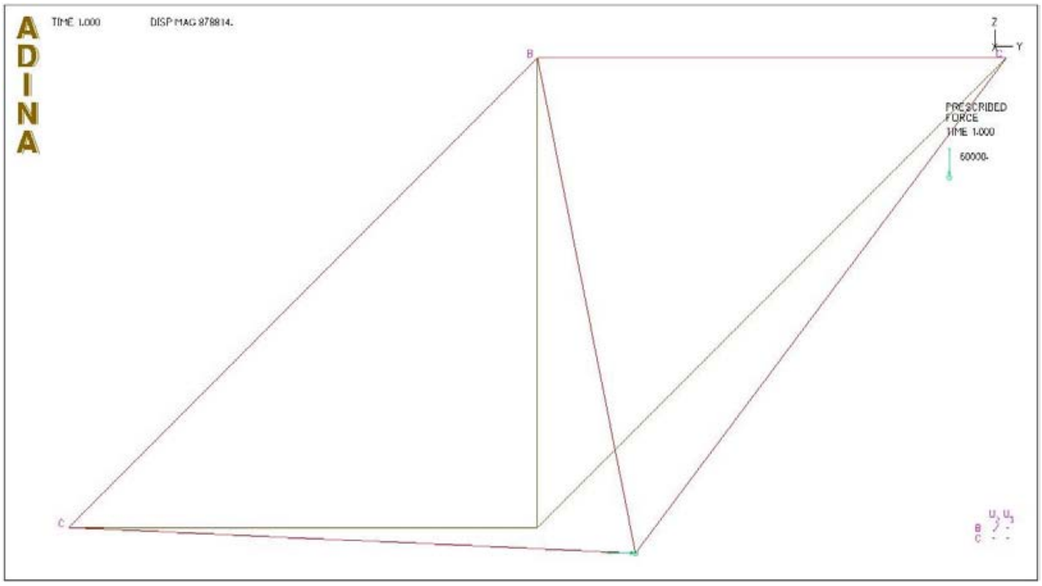
The solution is

$$\begin{bmatrix} U_3 \\ U_5 \\ U_6 \end{bmatrix} = \frac{a}{EA}\begin{bmatrix} 0 \\ 4.76\times10^4 \\ -1.24\times10^4 \end{bmatrix}\times10^4$$

c) Since we know the values of U_3, U_5, and U_6, we can calculate the reaction forces from $\text{K}_{\text{ba}}\mathbf{U}_{\text{a}}\mathbf{=R}_{\text{b}}$,

$$\begin{bmatrix} R_1 \\ R_2 \\ R_4 \\ R_7 \\ R_8 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 0 & -1 \\ -\frac{1}{2\sqrt{2}} & -1 & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 4.76\times10^4 \\ -1.24\times10^4 \end{bmatrix} N = \begin{bmatrix} -1.24\times10^4 \\ -1.24\times10^4 \\ 1.24\times10^4 \\ -4.76\times10^4 \\ 0 \end{bmatrix} N = \begin{bmatrix} R_1 \\ R_2 \\ R_4 \\ R_7 \\ R_8 \end{bmatrix}$$

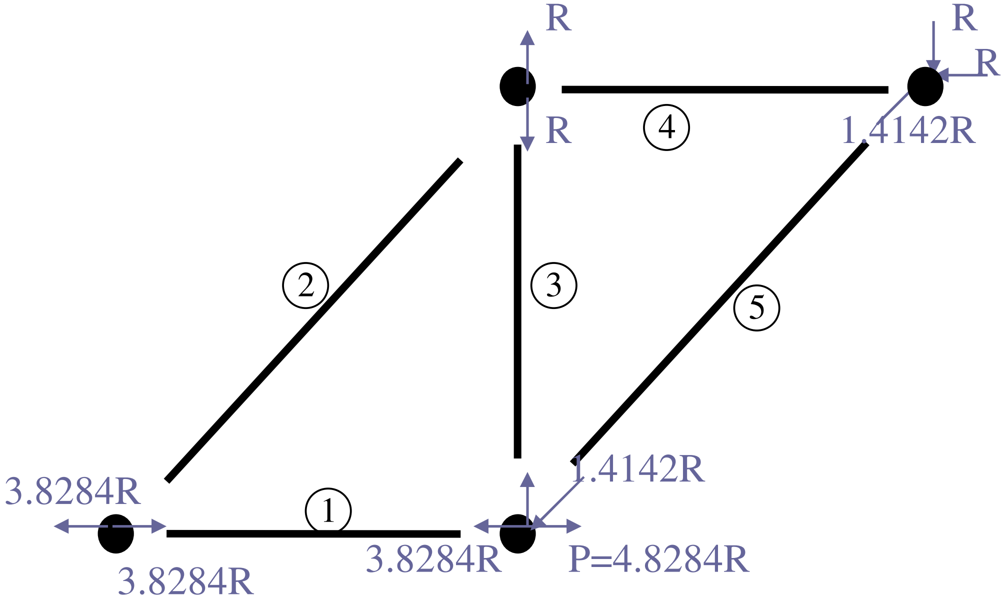
The undeformed and deformed meshes with applied boundary conditions and loads are plotted on the next page.



To calculate all internal forces, let's draw the equilibrium diagram for each joint.

$R=1.2426\times10^4$ N

$P=6\times10^5$ N



Therefore, the internal forces are

Element 1: tension 4.7572×10^4 N

Element 2: no force

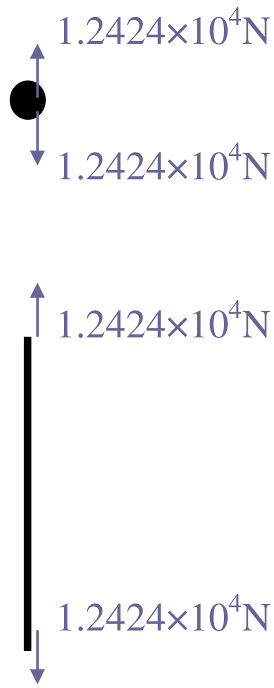
Element 3: tension 1.2426×10^4 N

Element 4: no force

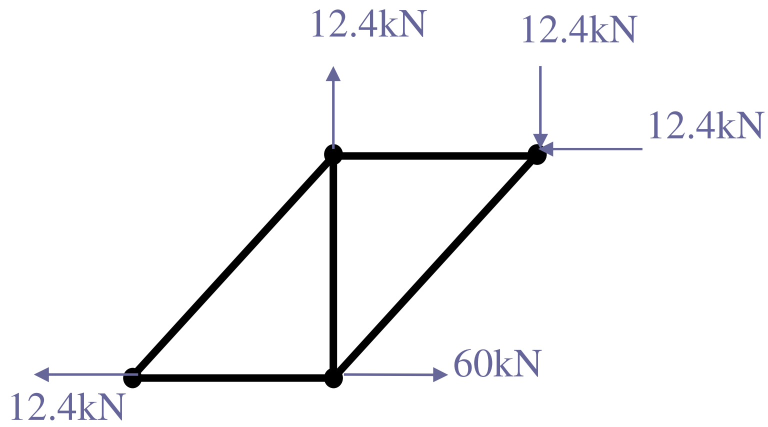
Element 5: Compression 1.7573×10^4 N

and Reactions: $$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \end{bmatrix} = \begin{bmatrix} -1.24\times10^4 \\ -1.24\times10^4 \\ 0 \\ 1.24\times10^4 \\ 6\times10^5 \\ 0 \\ -4.76\times10^4 \\ 0 \end{bmatrix} N$$

d) We can make sure that element 3 and joint 3 are in equilibrium explicitly by the diagram below.



External forces acting on structure.



$$\sum \vec{F}_x = -12.4kN - 47.6kN + 60kN = 0$$

$$\sum \vec{F}_y = 12.4kN - 12.4kN = 0$$

$$\sum \vec{M}_{at4} = -12.4kN \times a - 47.6kN \times a + 60kN \times a = 0$$

Therefore, the structure is in equilibrium.

Question: Why does the joint 3 not move horizontally? Assume you have not calculated the detailed solution given above. Give your answer and a physical reason.