

Homework #4

Date Issued: Wednesday 1 October, 2014

Date Due: Wednesday 8 October, 2014, 9:30AM (bring hard copy to lecture)

As described in the course policies document, this is one of 5 homeworks you will complete in this course. Each of these count as 6% of your total grade. Full credit can generally only be earned by showing your work. This often includes making clear and well-labeled plots.

1) (50 points)

A cylindrical buoy of radius r , height h and density ρ_b is released from rest with its base touching the surface of the ocean with water of density ρ_w . Thereafter, it is acted upon by two forces: a downward gravitational force equal to its weight, $\rho_b \pi r^2 h g$, and an upward buoyancy force equal to the weight of water displaced, $\rho_w \pi r^2 x g$, where $x(t)$ is the depth of the bottom of the buoy beneath the ocean surface at time t .

- Provided that $\rho_b < \frac{1}{2}\rho_w$, show that the buoy undergoes simple harmonic motion about an equilibrium position and obtain expressions for both the frequency of the oscillation and the equilibrium position.
- When $\rho_b > \frac{1}{2}\rho_w$ something breaks down with the model, which can no longer be used to describe the motion. Can you identify the problem?
- Finally, suppose the $\rho_b > \rho_w$ and the buoy is initially completely submerged beneath the surface (i.e. $x(0) = h, x'(0) = 0$). Please solve for the resulting position of the buoy as a function of time.

2) (20 points) Suppose a predator eats a prey that eats a smaller prey. The dynamics of that population might be modeled by the non-linear system of differential equations below:

$$dx/dt = -x + xy$$

$$dy/dt = -xy + y + yz$$

$$dz/dt = -yz + 2z$$

- Find all the critical points.
- Compute the Jacobian matrix at all the critical points and assess the stability of the critical pts.

3) (30 points) Draw a statically determinate truss structure with at least 5 members. Set up a linear system of equations that will allow you to find the tension or compression in the members and the reactions at the supports given known loads at the pinned joints. Solve the system and make sure the answers make sense for at least one loading condition.

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2.087 Engineering Math: Differential Equations and Linear Algebra
Fall 2014

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