2.087 Differential Equations and Linear Algebra, Spring 2014

Homework #3

Date Issued: Friday 19 September, 2014

Date Due: Wednesday 1 October, 2014, 9:30AM (bring hard copy to lecture)

As described in the course policies document, this is one of 5 homeworks you will complete in this course. Each of these count as 6% of your total grade. Full credit can generally only be earned by showing your work. This often includes making clear and well-labeled plots.

1) (20 points) Find the general solution to the following ordinary differential equations.

a. $6y'' - y' - y = 0$

b. $y'' - 6y' + 19y = 0$

c. $y'' + 5y' = 0$

2) (15 points)

The equation governing the motion of a pendulum of length L is:

$$L\theta'' + g\theta = 0,$$

where $g = GM/R^2$ is the gravitational acceleration at the location of the pendulum (at distance R from the center of the Earth and M is the mass of the Earth). A certain pendulum clock keeps perfect time in Paris, where the radius of the Earth is $R = 3956$ miles. But this clock loses 2 mins and 40 seconds per day at a location on the equator. Use this measurement to determine the radius of the Earth at the equator.

3) (15 points)

Solve the initial value problem $y'' - y' - 2y = 0$, $y(0) = \alpha$, $y'(0) = 2$. Then find α so that the solution approaches zero as $t \to \infty$.

4) (10 points) Show that $y = \int_0^t g(t-s) f(s) ds$ is a solution to $my'' + ky = f(t)$

a) Why is $y' = \int_0^t g'(t-s) f(s) ds + g(0) f(t)$

b) Using $g(0) = 0$ explain why $y'' = \int_0^t g''(t-s) f(s) ds + g'(0) f(t)$

c) Now use $g'(0) = 1/m$ and $mg'' + kg = 0$ to confirm $my'' + ky = f(t)$

5) (10 points) Substitute $y = e^{st}$ and solve the characteristic equation for s.

a) $2y'' + 8y' + 6y = 0$

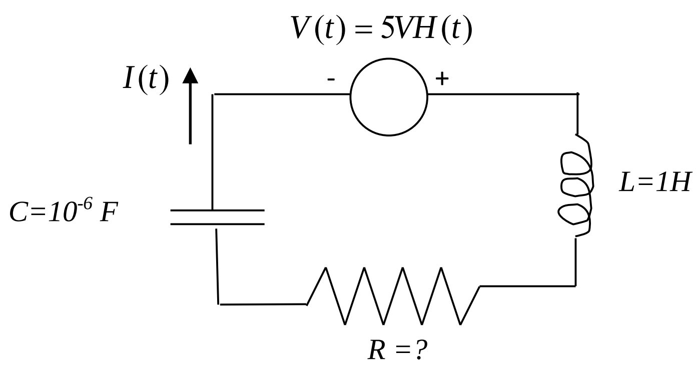
b) $y'''' + y'' + y = 0$

6) (15 points) Solve these differential equation which have exponential driving force

a) $y'' + 3y' + 5y = e^t$

b) $2y'' + 4y = e^{it}$

7) (15 points) The figure depicts an RLC circuit with a step response



a) For what values of the unknown R is the circuit is underdamped?

b) Solve for the current $I(t)$ in the circuit in terms of the unknown R.

c) If the circuit is underdamped, the current is at first positive, then reverses at some point (at least once) and overshoots past the steady state (in this case, I_{∞}=0). Find (or estimate) the resistance R that will produce an overshoot of -0.2A beyond the steady state response (but not a larger overshoot).