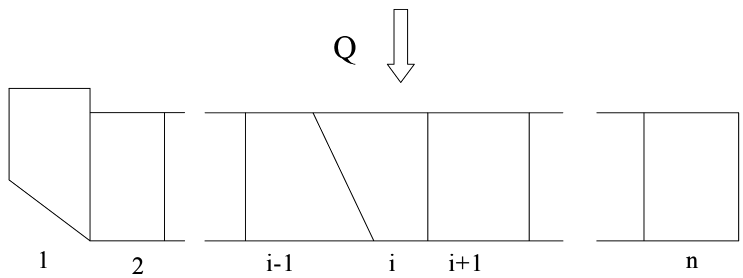
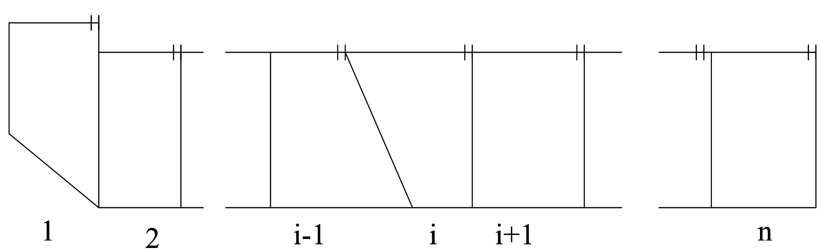
Shear stress due to Shear load (pure bending) multi-cell closed cross-section



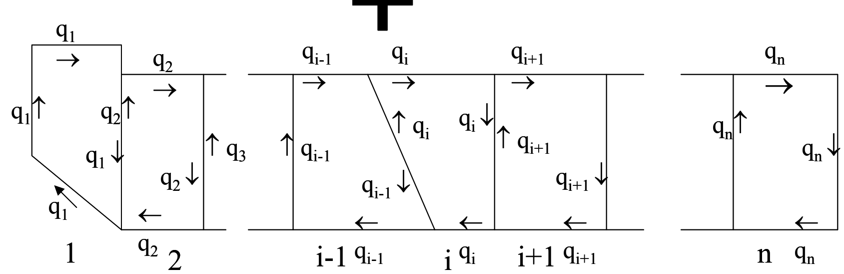
With resulting distribution of shear flow q(x,y) or q(s)

=



open section with shear flow q*(s)

+



n closed loop (constant) shear flows in each closed cell

for open section portion:

$$\text{m\_star}(s) = \int_0^s y(s)\cdot t(s)\, ds \qquad \text{q\_star}(s) = \frac{Q}{I}\cdot \text{m\_star}(s)$$

IF Iyz = 0
remember y(s) is distance in y direction from centroid

otherwise:

$$\text{m\_star}_y(s) = \int_0^s y(s)\cdot t(s)\, ds \qquad \text{m\_star}_z(s) = \int_0^s z(s)\cdot t(s)\, ds$$

$$q\_star(s) = \frac{V_y(x)}{\left(-I_{yz}^{\ 2} + I_{yy} \cdot I_{zz}\right)} \cdot \left(I_{yz}\, m\_star_z(s) - I_{yy}\, m\_star_y(s)\right)$$

$$q\_cell_i(s) = q\_star\_cell_i(s) + \sum_{(i,k)} q_{ik} \qquad q_{ik} = q_i - q_k$$

1 = 1, 2, 3, ..., n
k = adjacent cell to i (i+1 and i-1 in above figure)
= qi where no adjacent cell

$$\int \gamma\, ds = 0$$

1 = 1, 2, 3, ..., n
integral is circular
this is condition of no slip

$$\int \gamma\, ds = \frac{1}{G} \cdot \int \tau\, ds = \frac{1}{G} \cdot \int \frac{q}{t}\, ds = 0 \quad \text{as} \quad \tau = \frac{q}{t} \qquad G \neq 0$$

=> for each cell i

$$0 = \int \frac{q}{t}\, ds = \sum_{cell_i} \int q_i \frac{1}{t}\, ds - \sum_{common\_side\_i\_k} \left(\left(\int q_k \frac{1}{t}\, ds \right) + \int \frac{q\_star_i}{t}\, ds \right.$$

where integration is summed over each wall element (circular integral in q_star case)

but since qi and qk are constant over all walls it can be extracted from the sum =>

$$q_i \cdot \int \frac{1}{t}\, ds - \sum_k q_k \cdot \int \frac{1}{t}\, ds = -\int \frac{q\_star_i}{t}\, ds$$

this is a system of n linear equations:

first and rhs integrals are circular whereas second is over wall common to i and k and $\int \frac{q\_star_i}{t}\, ds$ is the integral of the open shear flow around cell i

for example cell 1 lhs
cell 1 cell 2

$$q_1 \cdot \int \frac{1}{t}\, ds - q_2 \cdot \int \frac{1}{t}\, ds$$

1,2 => integral along common wall of 1 and 2
1 => circular integral around cell 1

and entire system of equations becomes:

1 1,2

$$q_1 \cdot \int \frac{1}{t}\,ds - q_2 \cdot \int \frac{1}{t}\,ds = -\int \frac{q\_star_1}{t}\,ds$$

1 1,2

$$-q_1 \cdot \int \frac{1}{t}\,ds + q_2 \cdot \int \frac{1}{t}\,ds - q_3 \cdot \int \frac{1}{t}\,ds = -\int \frac{q\_star_2}{t}\,ds$$

1,2 2 2,3

$$-q_2 \cdot \int \frac{1}{t}\,ds + q_3 \cdot \int \frac{1}{t}\,ds - q_4 \cdot \int \frac{1}{t}\,ds = -\int \frac{q\_star_3}{t}\,ds$$

2,3 3 3,4

.... = ...

$$q_{n-1} \cdot \int \frac{1}{t}\,ds - q_n \cdot \int \frac{1}{t}\,ds = -\int \frac{q\_star_n}{t}\,ds$$

n-1,n n

let each element of the matrix $\int \frac{1}{t}\,ds$ be expressed by η

where $\eta_{ik} = \int \frac{1}{t}\,ds$ integral along wall separating i and k

i,k

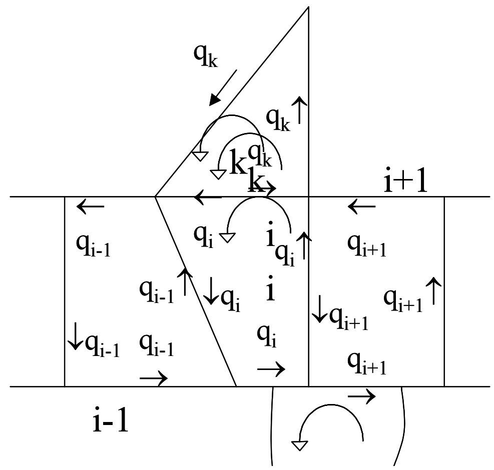
and $\eta_{ii} = \int \frac{1}{t}\,ds$ integral around cell i

if wall thickness is piecewise constant walls =>

$$\eta_{ik} = \frac{s_{ik}}{t_{ik}} \quad \text{and} \quad \eta_{ii} = \sum_{j=1}^{4} \frac{s_{ij}}{t_{ij}} = \frac{s_{i1}}{t_{i1}} + \frac{s_{i2}}{t_{i2}} + \frac{s_{i3}}{t_{i3}} + \frac{s_{i4}}{t_{i4}}$$

where s_{ij}, t_{ij} is the length and thickness of wall j of cell i

we observe that the matrix is symmetric i.e. $\eta_{ik} = \eta_{ki}$ for the figure we are analyzing the adjacency can be general as shown:



with contribution =>

$$q_i \cdot \eta_{ii} - q_{i-1} \cdot \eta_{i-1,i} - q_k \cdot \eta_{ki} - q_{i+1} \cdot \eta_{i,i+1} - q_m \cdot \eta_{im} = -\int \frac{q\_star_i}{t}\, ds$$

if t not constant $\eta_{ik} = \int \frac{1}{t(s)}\, ds$ along wall between i and k

we now have n equations and n unknowns; qi

solution:

first calculate q_star from m_star (it's tedious) then

calculate $\eta_{ik} = \frac{s_{ik}}{t_{ik}}$ or $\int \frac{1}{t(s)}\, ds$ along wall between i and k

calculate $\eta_{ii} = \sum_{j=1}^{4} \frac{s_{ij}}{t_{ij}} = \frac{s_{i1}}{t_{i1}} + \frac{s_{i2}}{t_{i2}} + \frac{s_{i3}}{t_{i3}} + \frac{s_{i4}}{t_{i4}}$ or $\int \frac{1}{t(s)}\, ds$ around cell i

check problem on page 117 Hughes with one additional cell added: new sheet