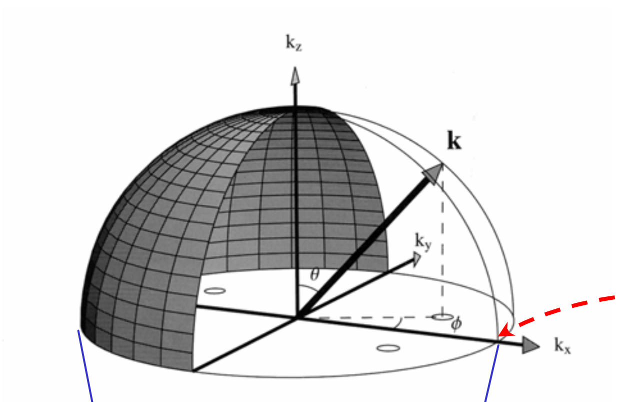
13.811
Advanced Structural Dynamics and Acoustics

Acoustics
Lecture 5

Ewald Sphere Construction
Baffled Piston



Directivity Function

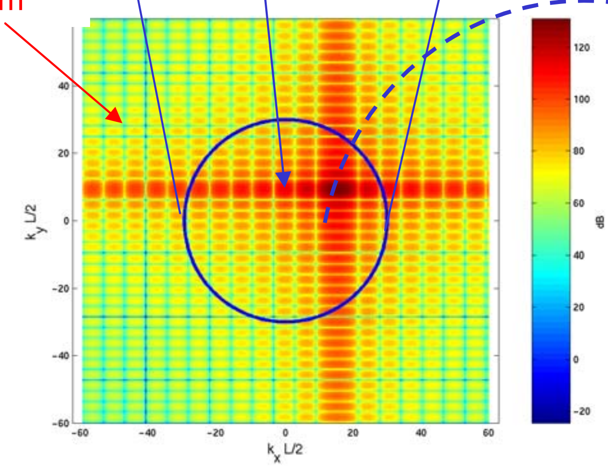
$$\boxed{D(\theta,\phi) = -\frac{i\rho\omega}{2\pi}\; \dot{w}_\omega(k_x,k_y;0)}$$

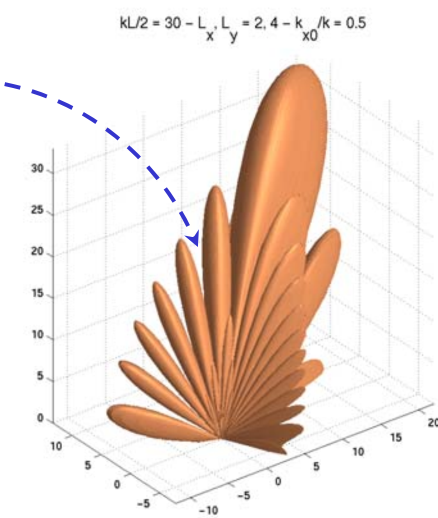
$$f = \omega/2\pi = kc/2\pi$$

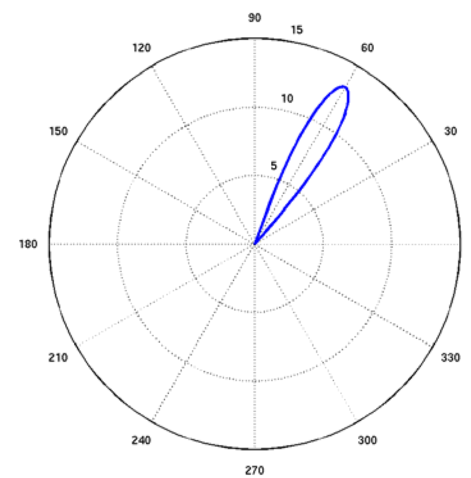
$$\begin{aligned} k_x &= k \sin\theta cos\phi \\ k_y &= k \sin\theta sin\phi \end{aligned}$$

Radiating Spectrum

Evanescent Spectrum





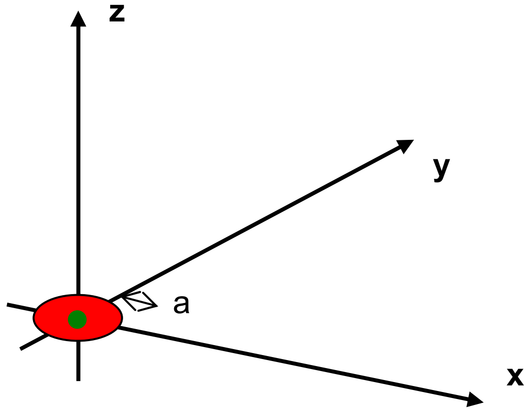


circ.m

```
%
% MATLAB script for plotting the directivity function for
% a circular, baffled piston
%
% Parameters:
% k     Wavenumber
% rho   Density
% c     Speed of Sound
% a     Radius of piston of piston
%
clear
figure(1)
hold off
k=10;
rho=1000;
c=1500;

a=1.0;


ka=k*a;

figure(1);
kxm=2*ka;
nkx=300;
dkx=2*kxm/(nkx-1);
x=[-kxm:dkx:kxm];
y=x;
o=ones(1,nkx);
kx=x' * o;
ky=(y' *o)';
kr=abs(complex(kx,ky));
ss=rho*k*c*a^2 * besselj(1,kr)./kr;
%surfc(kx,ky,dba(ss));
wavei(dba(ss)',x,y)
shading('flat')
axis('equal')
b=xlabel('k_x a')
set(b,'FontSize',16);
b=ylabel('k_y a')
set(b,'FontSize',16);
tit=['Circular Piston - ka = ' num2str(k*a) ]
b=title(tit);
set(b,'FontSize',20);
nphi=361;
dphi=2*pi/(nphi-1);
phi=[0:dphi:2*pi];
xx=k*a*cos(phi);
yy=k*a*sin(phi);
hold on
b=plot(xx,yy,'b');
set(b,'LineWidth',3);
```

```
figure(2)
nphi=361.
dphi=2*pi/(nphi-1)
nth=181;
dth=0.5*pi/(nth-0.5);

phi=[0:dphi:(nphi-1)*dphi]' * ones(1,nth);
th=([dth/2:dth:pi/2]'*ones(1,nphi))';
kx=ka*sin(th).*cos(phi);
ky=ka*sin(th).*sin(phi);
kr=ka*sin(th);
ss=rho*k*c*a^2*besselj(1,kr)./kr;

ss=dba(ss);
sm=max((max(ss))');
for i=1:size(ss,1)
     for j=1:size(ss,2)
     ss(i,j)=max(ss(i,j),sm-40.0)-(sm-40.0);
end
end

xx=ss.*sin(th).*cos(phi);
yy=ss.*sin(th).*sin(phi);
zz=ss.*cos(th);

surfl(xx,yy,zz);
colormap('copper');
shading('flat');
axis('equal');
tit=['Circular Piston - ka = ' num2str(k*a) ]
b=title(tit);
set(b,'FontSize',20);


figure(3)
b=polar([pi/2-fliplr(th(1,:)) pi/2+th((nphi-1)/2+1,:)],[fliplr(ss(1,:))
ss((nphi-1)/2+1,:)]);
set(b,'LineWidth',2)

tit=['Circular Piston - ka = ' num2str(k*a) ]
b=title(tit);
set(b,'FontSize',20);
```

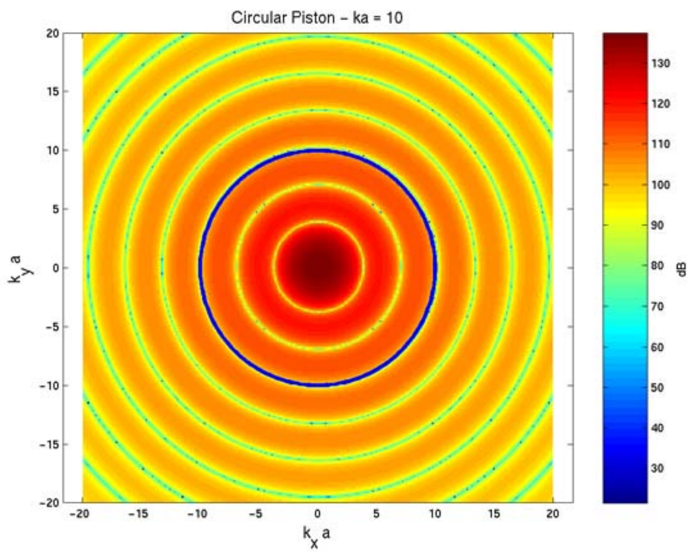
Circular Piston

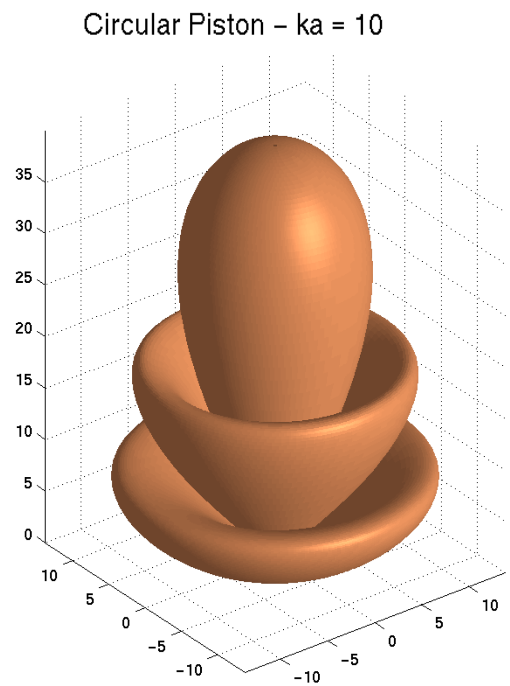


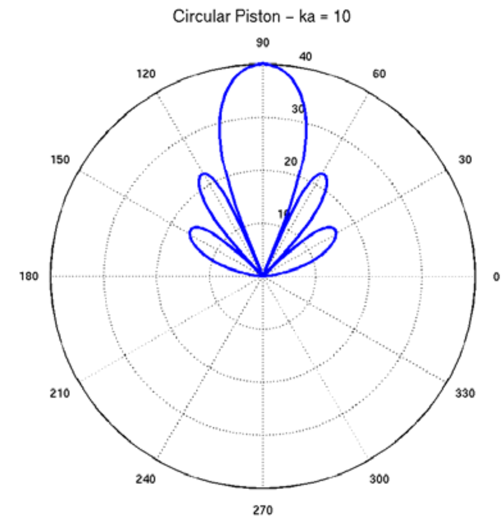
Directivity Function

$$D(\theta,\phi) = D(\theta) \;=\; -i\rho\omega a^2 \frac{J_1(k_r a)}{k_r a}$$

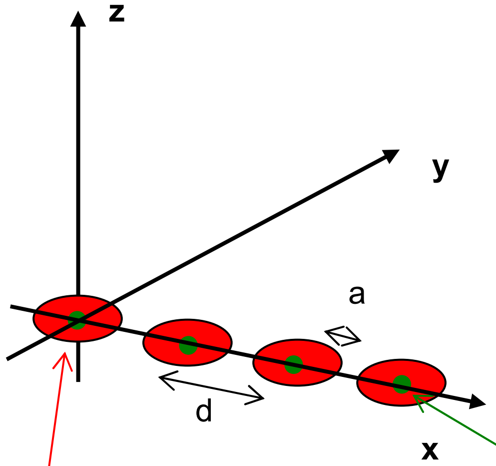
$$f = \omega/2\pi = kc/2\pi$$







Array of Baffled Pistons



Baffled Piston Array

$$\dot{w}_n(x, y, 0) = S_n \dot{w}(x - x_n, y - y_n, 0)$$

Fourier Transform

$$\dot{w}_n(k_x, k_y, 0) = S_n \dot{w}(k_x, k_y, 0) e^{-ik_x x_n} e^{ik_y y_n}$$

Directivity Function

$$D(\theta, \phi) = -\frac{i\rho\omega}{2\pi} \dot{w}_n(k_x, k_y, 0)$$

$$= -\frac{i\rho\omega}{2\pi} \dot{w}(k_x, k_y, 0) \sum_{n=1}^{N} S_n e^{-ik_x x_n} e^{-ik_y y}$$

One Baffled Piston

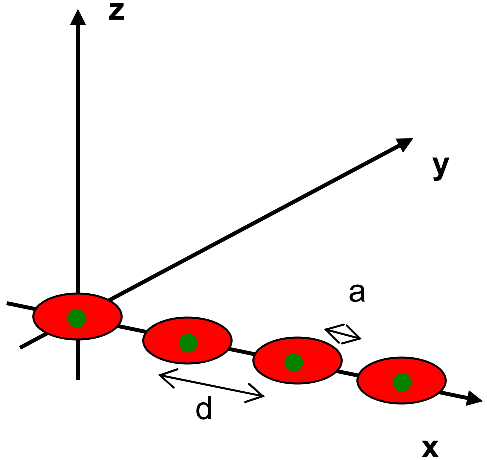
Array of Point Sources

$$= \boxed{-\frac{i\rho\omega}{2\pi} \dot{w}(k_x, k_y, 0)} \boxed{\sum_{n=1}^{N} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_n \delta(x - x_n) \delta(y - y_n) e^{-ik_x x} e^{-ik_y y} dx dy}$$

circ_arr.m

```
%
% MATLAB script for plotting the directivity function for
% an array of circular, baffled pistons
%
% Parameters:
% k      Wavenumber
% rho    Density
% c      Speed of Sound
% a      Radius of piston of piston
% d      Piston separation
% nd     Number of pistons
% qd     Array of piston strengths

clear
figure(1)
hold off
k=10.0;
rho=1000;
c=1500;
a=1.0;

% Half wavelength spacing, d= pi/k
d=pi/k;
nd=10;
ah=(nd-1)*d/2
xd=[-ah:d:ah]';
kxd_0=k/2;
qd=ones(length(xd),1);
qd=exp(-i*kxd_0*xd);
%qd(2)=-1;
ka=k*a;
figure(1);
kxm=2*ka;
nkx=300;
dkx=2*kxm/(nkx-1);
x=[-kxm:dkx:kxm];
y=x;
o=ones(1,nkx);
kx=x' * o;
ky=(y' *o)';
kr=abs(complex(kx,ky));
kx1=reshape(kx,1,nkx^2);
shd=qd'*exp(-i*xd*kx1);
shd=reshape(shd,nkx,nkx);
ss=rho*k*c*a^2 * besselj(1,kr)./kr;
wavei(dba(ss.*shd)',x,y)
shading('flat')
axis('equal')
b=xlabel('k_x a')
set(b,'FontSize',16);
b=ylabel('k_y a')
set(b,'FontSize',16);
tit=['Circular Piston Array - ka = ' num2str(k*a) ' - d,n =' num2str(d)
',  '  num2str(nd) ]
b=title(tit);
set(b,'FontSize',20);nphi=361;
dphi=2*pi/(nphi-1);
phi=[0:dphi:2*pi];
xx=k*a*cos(phi);
yy=k*a*sin(phi);
hold on
b=plot(xx,yy,'b');
set(b,'LineWidth',3);

figure(2)
nphi=361.
dphi=2*pi/(nphi-1)
nth=181;
dth=0.5*pi/(nth-0.5);

phi=[0:dphi:(nphi-1)*dphi]' * ones(1,nth);
th=([dth/2:dth:pi/2]'*ones(1,nphi))';
kx=ka*sin(th).*cos(phi);
ky=ka*sin(th).*sin(phi);
kr=ka*sin(th);
ss=rho*k*c*a^2*besselj(1,kr)./kr;
kx1=reshape(kx,1,size(kx,1)*size(kx,2));
shd=qd'*exp(-i*xd*kx1);
shd=reshape(shd,size(kx,1),size(kx,2));
ss=dba(ss.*shd);
sm=max((max(ss))');
for i=1:size(ss,1)
 for j=1:size(ss,2)
  ss(i,j)=max(ss(i,j),sm-40.0)-(sm-40.0);
 end
end
xx=ss.*sin(th).*cos(phi);
yy=ss.*sin(th).*sin(phi);
zz=ss.*cos(th);
surfl(xx,yy,zz);
colormap('copper');
shading('flat');
axis('equal');
tit=['Circular Piston Array - ka = ' num2str(k*a) ' - d,n =' num2str(d)
', ' num2str(nd) ]
b=title(tit);
set(b,'FontSize',20);
```

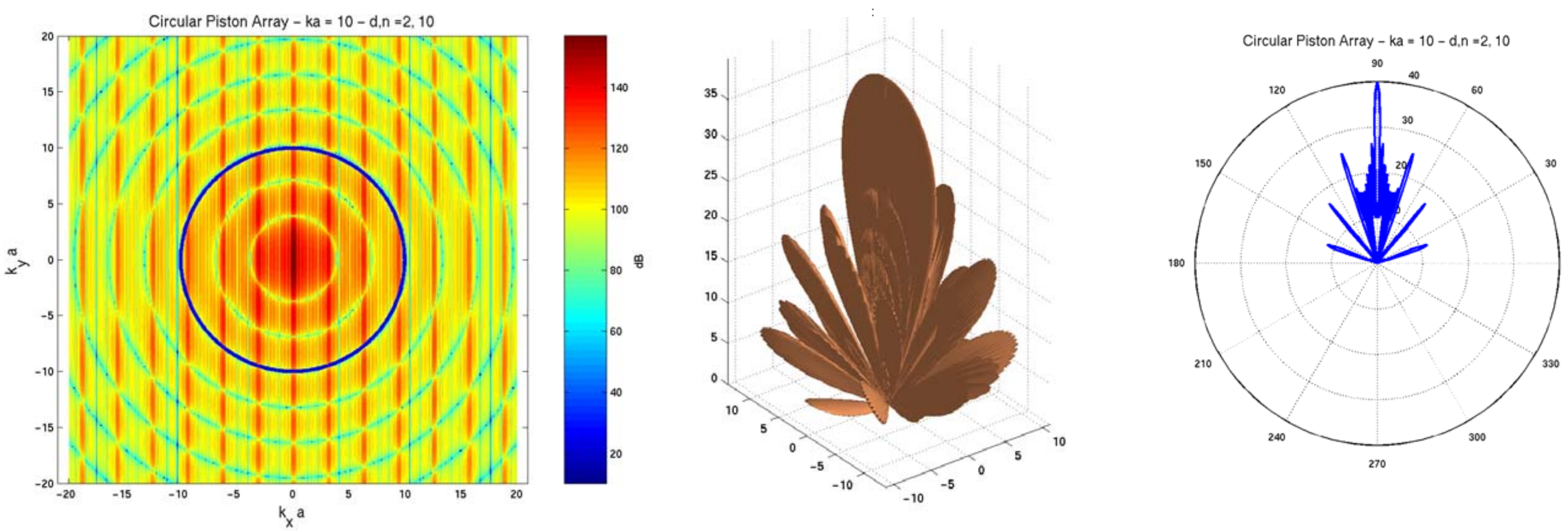
Array of Circular Pistons



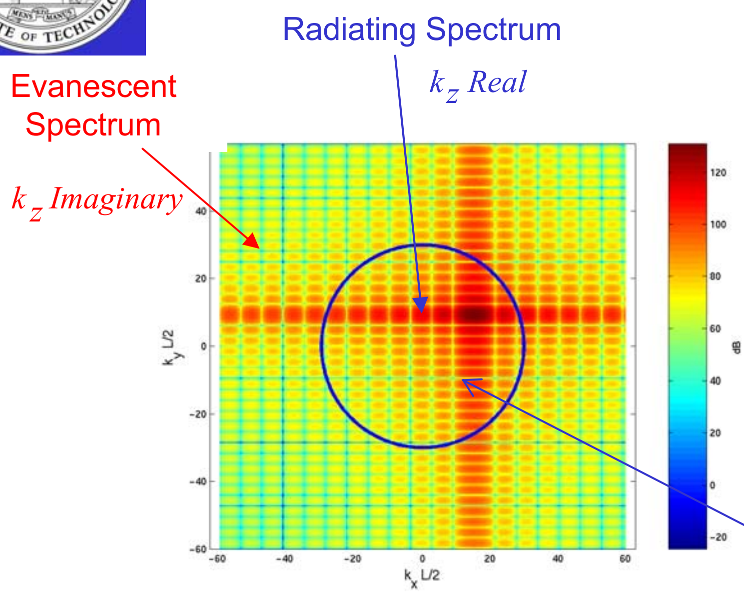
Directivity Function

$$
\begin{aligned}
D(\theta,\phi) &= -\frac{i\rho\omega}{2\pi}\sum_{n=1}^{N}\dot{w}_n(k_x,k_y,0) \\
&= -\frac{i\rho\omega}{2\pi}\dot{w}(k_x,k_y,0)\sum_{n=1}^{N} S_n e^{-ik_x x_n} e^{-ik_y y_n} \\
&= -\frac{i\rho\omega}{2\pi}\dot{w}(k_x,k_y,0)\sum_{n=1}^{N}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_n\delta(x-x_n)\delta(y-y_n)e^{-ik_x x}e^{-ik_y y}\,dx\,dy
\end{aligned}
$$

Circular Piston Array – ka = 10 – d,n =2, 10



Radiated Power



$$\Pi(\omega) = \frac{1}{8\pi^2}\mathrm{Re}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_\omega(k_x,k_y;0)\dot{w}_\omega(k_x,k_y;0)dk_xdk_y\right]$$

$$p_\omega(k_x,k_y;0) = \frac{\rho\omega}{k_z}\dot{w}_\omega(k_x,k_y;0)$$

Radiated Power

$$\Pi(\omega) = \frac{1}{2}\int\int_S Re[p_\omega(x,y,0)\dot{w}^*_\omega(x,y,0)]dS$$

Fourier Transforms

$$p_\omega(x,y,0) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_\omega(k_x,k_y;0)e^{i(k_xx+k_yy}dk_xdk_y$$

$$\dot{w}^*_\omega(x,y,0) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dot{w}_\omega(q_x,q_y;0)e^{-i(q_xx+q_yy}dq_xdq_y$$

$$\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(k_x-q_x)x}e^{i(k_y-q_y)y}dxdy = \delta(k_x-q_x)\delta(k_y-q_y)$$

$$\begin{aligned}\Pi(\omega) &= \frac{\rho\omega}{8\pi^2}\mathrm{Re}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{|\dot{w}_\omega(k_x,k_y;0)|}{k_z}dk_xdk_y\right]\\ &= \frac{\rho\omega}{8\pi^2}\int_k^k dk_x\int_{-\sqrt{k^2-k_x^2}}^{\sqrt{k^2-k_x^2}}\frac{|\dot{w}_\omega(k_x,k_y;0)|}{k_z}dk_y\end{aligned}$$

Point-Driven Plate Radiation

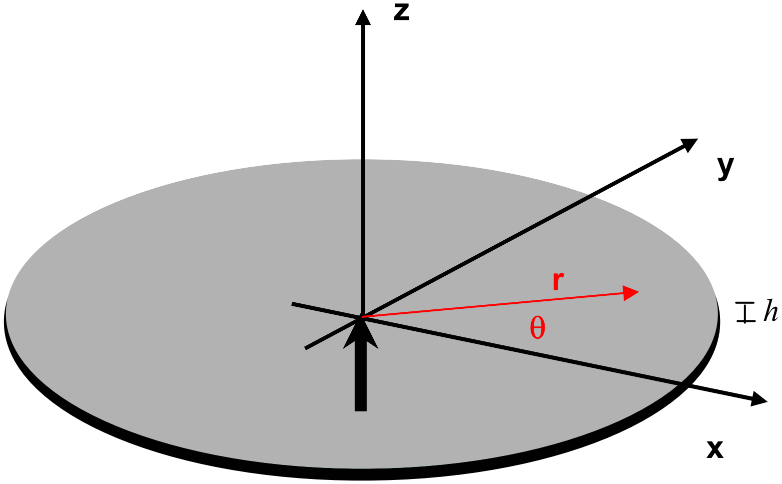


Plate Bending Equation

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho_s h \frac{\partial^2 w}{\partial t^2} = F(t)\delta(x)\delta(y) - p_a(x,y,t)$$

$$D = \frac{Eh^3}{12(1-\nu^2}$$

Skudrzyk's number

$$\alpha \equiv \left(\frac{D}{\rho_s h}\right)^{1/4} = \left(\frac{Eh^2}{12\rho_s(1-\nu^2)}\right)^{1/4}$$

Frequency Domain

$$D\left(\frac{\partial^4 w_\omega}{\partial x^4} + 2\frac{\partial^4 w_\omega}{\partial x^2 \partial y^2} + \frac{\partial^4 w_\omega}{\partial y^4}\right) + \rho_s h \omega^2 w_\omega = F_\omega \delta(x)\delta(y) - p_a(x,y)$$

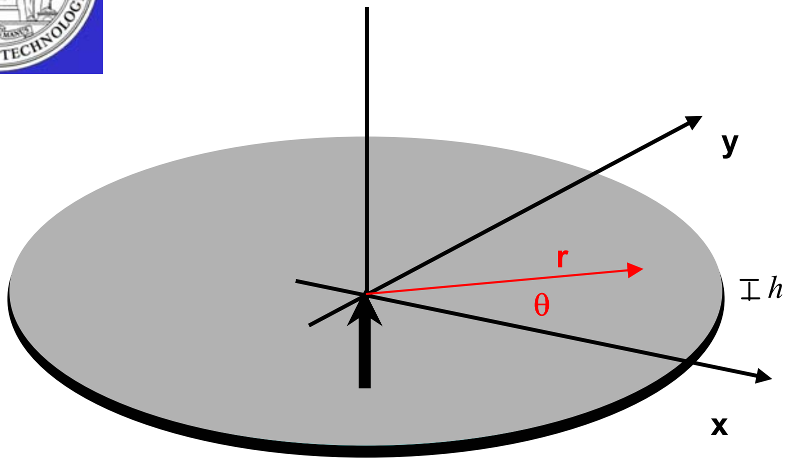
Cylindrical Coordinates

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r^2 &= x^2 + y^2 \end{aligned}$$

$$D\left(\frac{d^2}{dr^2} + \frac{d}{rdr}\right)^2 w_\omega - \rho_s h \omega^2 w_\omega = F_\omega \frac{\delta(r)}{2\pi r} - p_a(r)$$

$$\left(\frac{d^2}{dr^2} + \frac{d}{rdr}\right)^2 = \left(\frac{d^2}{dr^2} + \frac{d}{rdr}\right)\left(\frac{d^2}{dr^2} + \frac{d}{rdr}\right)$$

Point-Driven Plate Radiation



Hankel Transforms

$$\left(\frac{d^2}{dr^2}+\frac{d}{rdr}\right)J_0(k_r r) = -k_r^2 J_0(k_r r)$$

$$\frac{\delta(r)}{2\pi r} = \int_0^\infty J_0(k_r r)k_r dk_r$$

Light Fluid Loading

$$Dk_r^4 w_\omega(k_r) - \rho_s h\omega^2 w_\omega(k_r) \simeq F_\omega$$

$$w_\omega(k_r) = \frac{F(\omega)}{2\pi D(k_r^4 - k_f^4)}$$

Flexural Wavenumber

$$k_f = \left(\frac{m_s\omega^2}{D}\right)^{1/4} = \left(\frac{\rho_s h\omega^2}{D}\right)^{1/4}$$

$$k_f = \frac{\omega}{\alpha}$$

Particle Velocity

$$\dot{w}_\omega(k_r) = \frac{-i\omega F_\omega}{2\pi D(k_r^4 - k_f^4)}$$

Cylindrical Coordinates

$$D\left(\frac{d^2}{dr^2}+\frac{d}{rdr}\right)^2 w_\omega - \rho_s h\omega^2 w_\omega = F_\omega\frac{\delta(r)}{2\pi r} - \cancel{p_a(r)}$$

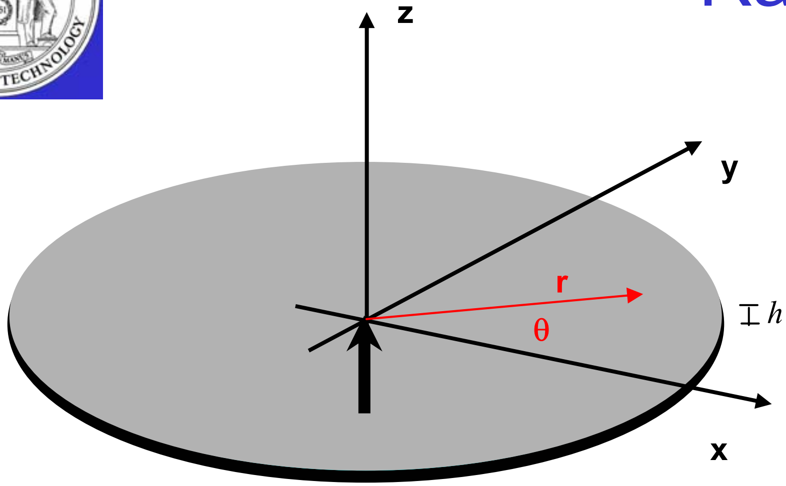
$$\left(\frac{d^2}{dr^2}+\frac{d}{rdr}\right)^2 = \left(\frac{d^2}{dr^2}+\frac{d}{rdr}\right)\left(\frac{d^2}{dr^2}+\frac{d}{rdr}\right)$$

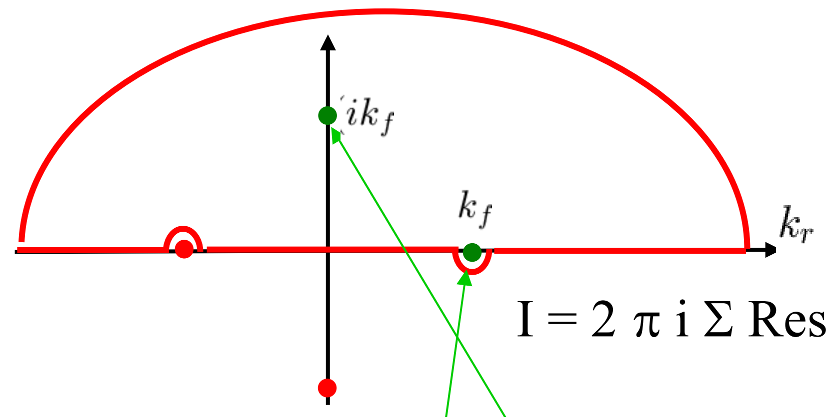
Hankel Transform

$$w(k_r) = \int_0^\infty w(r)J_0(k_r r)rdr$$

$$w(r) = \int_0^\infty w(k_r)J_0(k_r r)k_r dk_r$$

Radiated Field





Inverse Hankel Transform

$$w(r) = \frac{F_\omega}{2\pi D}\int_0^\infty \frac{J_0(k_r r)}{k_r^4 - k_f^4} k_r dk_r$$

$$= \frac{F_\omega}{2\pi D}\int_{-\infty}^\infty \frac{H_0^{(1)}(k_r r)}{k_r^4 - k_f^4} k_r dk_r$$

$$J_0(x) = \frac{1}{2}\left(H_0^{(1)}(x) + H_0^{(2)}(x)\right)$$

$$= \frac{1}{2}\left(H_0^{(1)}(x) - H_0^{(1)}(-x)\right)$$

Complex Contour Integration

$$\dot{w}_\omega(r) = \frac{F_\omega}{8\alpha^2 m_s}\left[\boxed{H_0^{(1)}(k_f r)} - \boxed{H_0^{(1)}(ik_f r)}\right]$$

$$= \frac{F_\omega}{8\alpha^2 m_s}\left[H_0^{(1)}(k_f r) - \frac{2i}{\pi}K_0(k_f r)\right]$$

$$H_0^{(1)}(k_r r) \rightarrow \sqrt{\frac{2}{\pi k_r r}}e^{i(k_r r - \pi/4)}$$

Drive-point Impedance

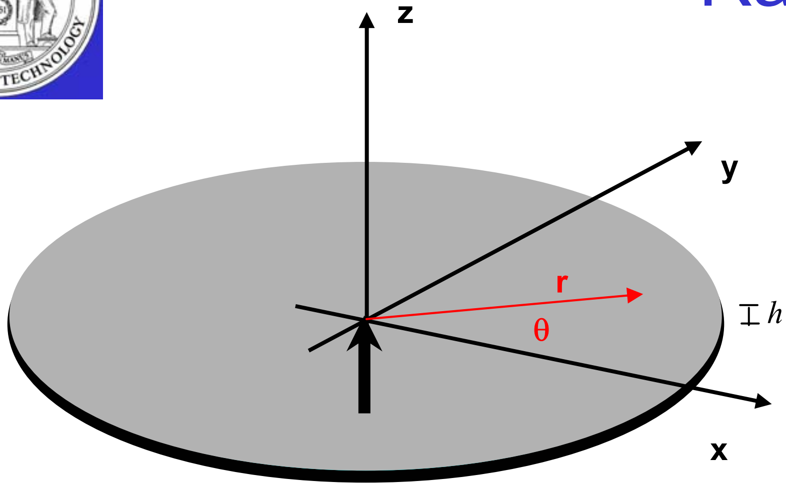
$$Z_p = F_\omega / \dot{w}(0) = 8\alpha^2 m_s$$

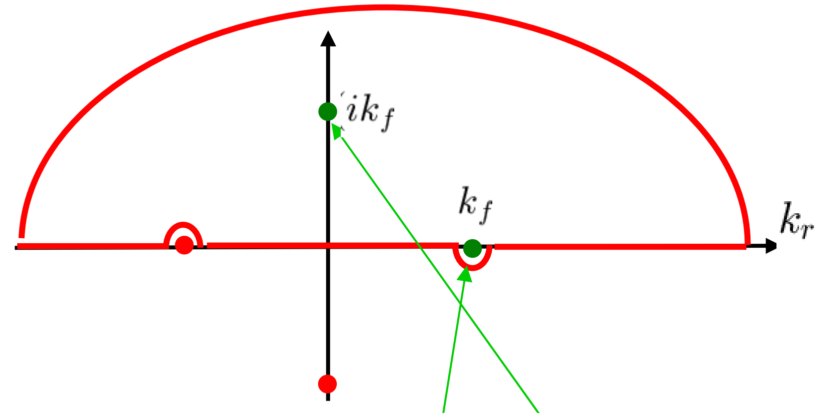
Flexural Wave Speed

$$k_f = \omega / c_f$$

$$c_f = \omega / k_f = \alpha\sqrt{\omega}$$

Radiated Field

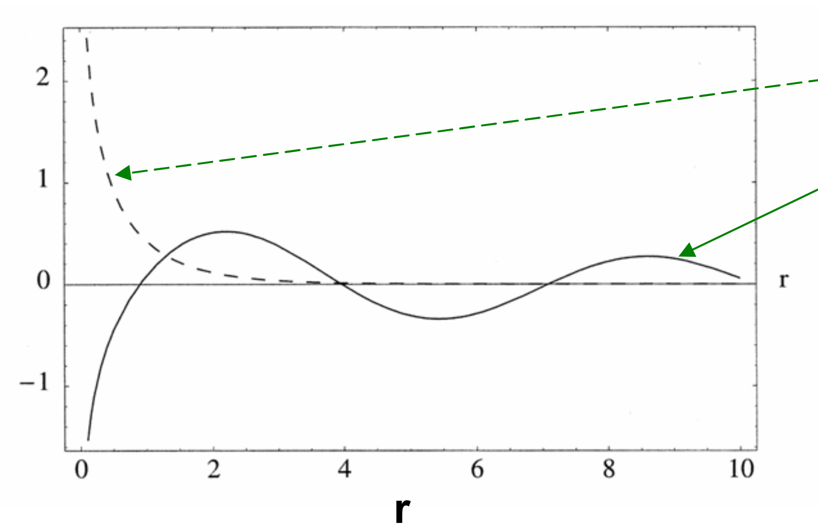




Complex Contour Integration

$$\dot{w}_\omega(r) = \frac{F_\omega}{8\alpha^2 m_s}\left[\boxed{H_0^{(1)}(k_f r)} - \boxed{H_0^{(1)}(ik_f r)}\right]$$

$$= \frac{F_\omega}{8\alpha^2 m_s}\left[H_0^{(1)}(k_f r) - \frac{2i}{\pi}K_0(k_f r)\right]$$



$$H_0^{(1)}(k_r r) \rightarrow \sqrt{\frac{2}{\pi k_r r}}e^{i(k_r r - \pi/4)}$$

Drive-point Impedance

$$Z_p = F_\omega/\dot{w}(0) = 8\alpha^2 m_s$$

Flexural Wave Speed

$$k_f = \omega/c_f$$

$$c_f = \omega/k_f = \alpha\sqrt{\omega}$$

Far Field Radiation

$$p(R,\theta,\phi) = -\frac{i\rho\omega}{2\pi}\frac{e^{ikR}}{R}\dot{w}_\omega(k_r) = -\frac{i\rho\omega}{2\pi}\frac{e^{ikR}}{R}\dot{w}_\omega(k sin\theta)$$

Image removed due to copyright considerations.
See Figure 2.23 in Williams, E. G. *Fourier Acoustics*.
London: Academic Press, 1999

$$\sin\theta_0 = k_f/k$$

$$k < k_f$$

Image removed due to copyright considerations.
See Figure 2.24 in [Williams].