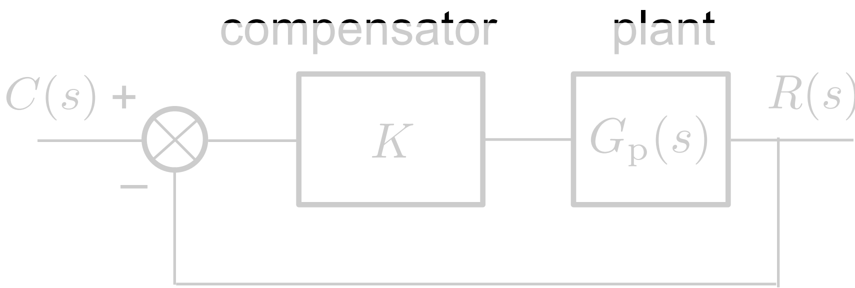
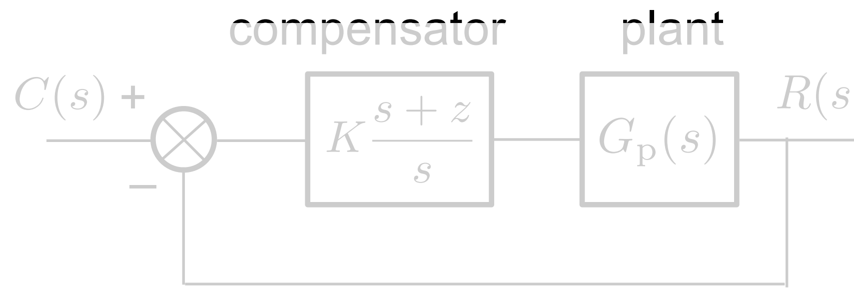
Classification of feedback compensators



Proportional (P)

can meet *one* transient specification (e.g. rise time)

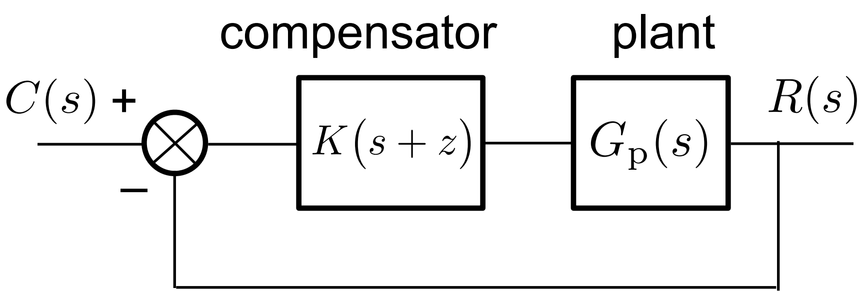
➡ other specifications (e.g. overshoot) unmet
➡ steady state error



Proportional-Integral (PI)

➡ can meet *one* specification (e.g. rise time)
➡ zero steady-state error

➡ other specifications (e.g. overshoot) unmet
➡ decreases stability (RL moves to the right)



Proportional-Derivative (PD)

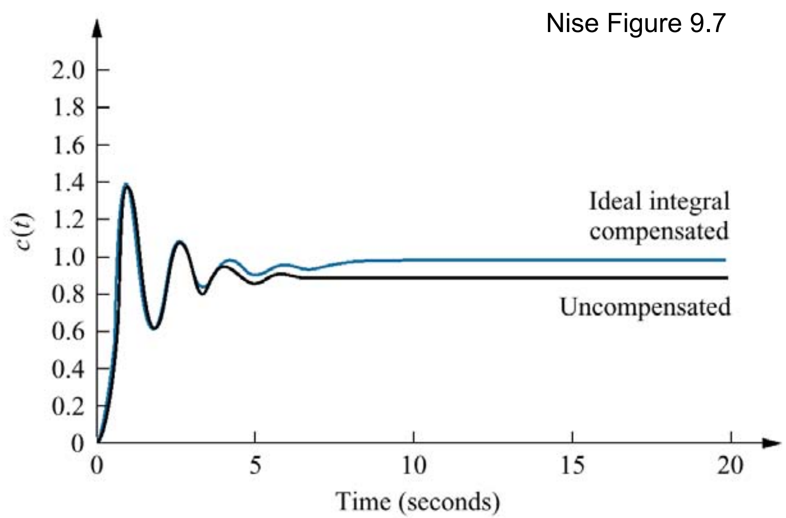
➡ speeds up response
➡ improves stability (RL moves to the left)

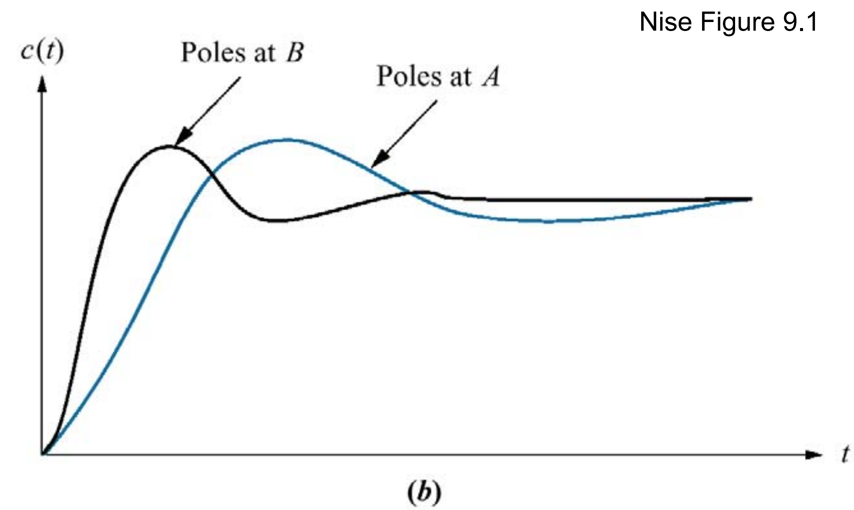
➡ may worsen steady state error
➡ noisy

Proportional-Integral-Derivative (PID)

➡ complete transient specification (rise time & overshoot)
➡ no steady-state error

Compensator rules of thumb

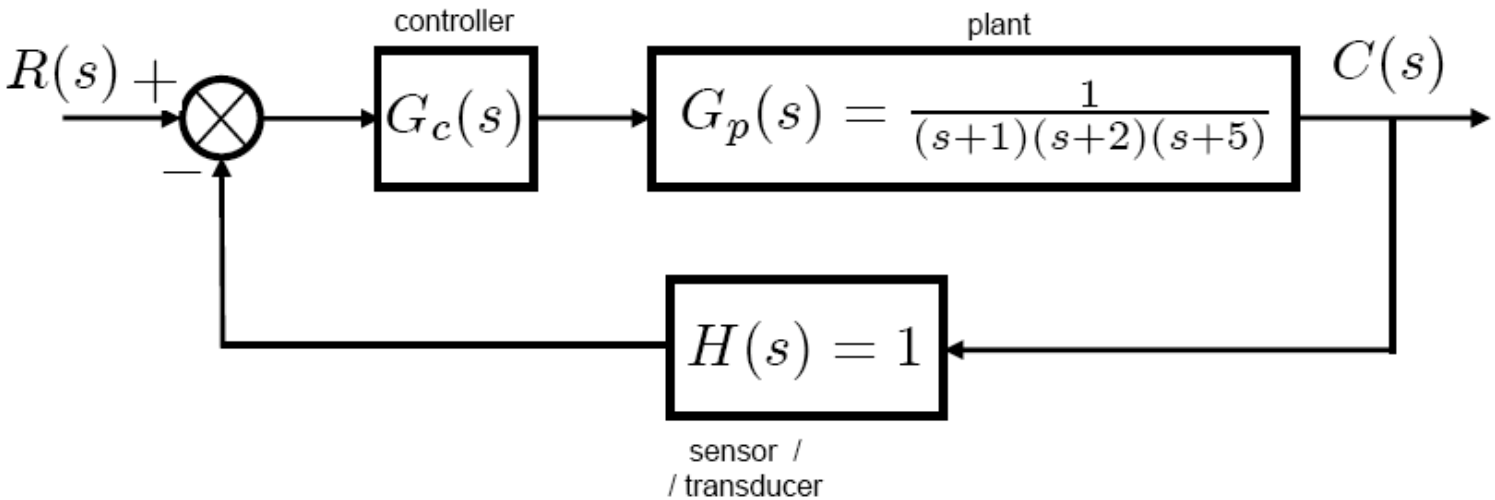




Integral action
- eliminates steady-state error; but,
- by itself, the integrator slows down the response;
  - therefore, a zero (derivative action) speeds the response back up to match the response speed of the uncompensated system

Derivative action
- speeds up the transient response;
- it *may* also improve the steady-state error; but
- differentiation is a ***noisy*** process
  - (we will deal with this later in two ways: the lead compensator and the PID controller)

Example



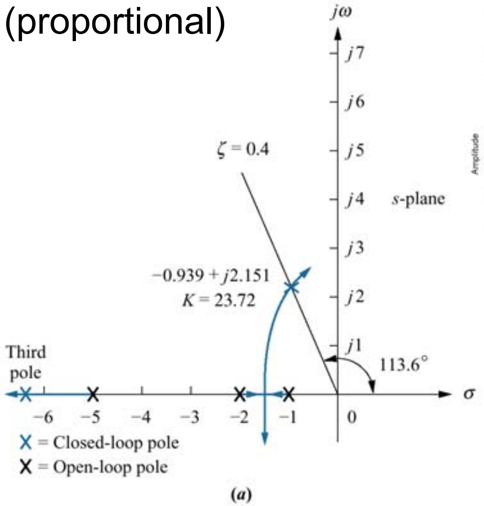
We wish to speed up the system response while maintaining $\zeta = 0.4 \Leftrightarrow \%OS \approx 25.4\%$.

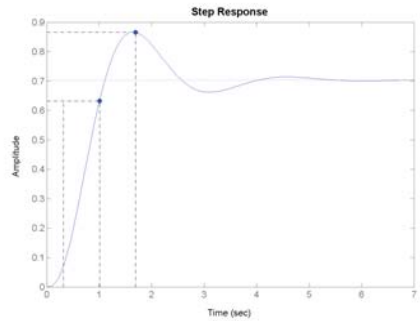
Evaluating different PD controllers

Nise Figure 9.15

$G_c(s) = K$

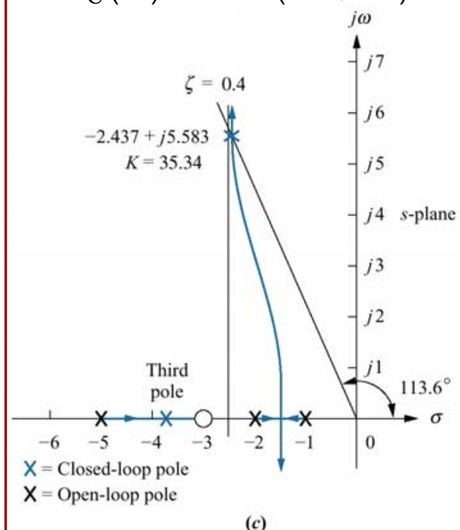
(proportional)

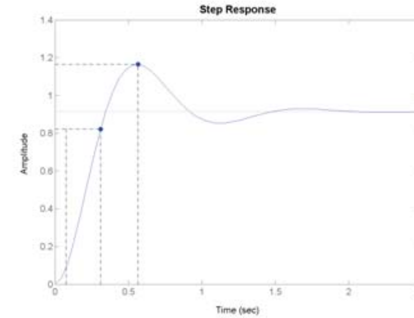




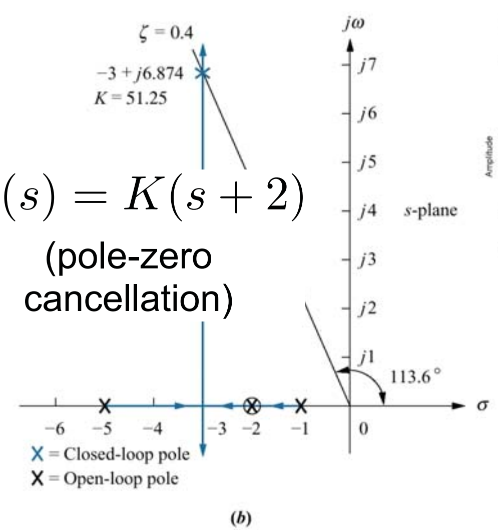
K=23.7
%OS=23.2
T_r=0.688 sec

$G_c(s) = K(s+3)$



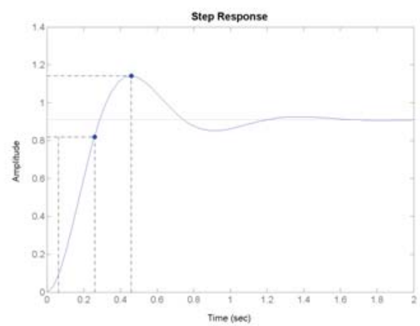


K=35.3
%OS=27.5
T_r=0.236 sec

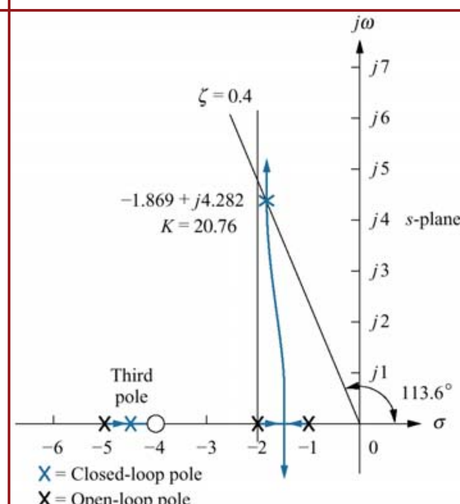


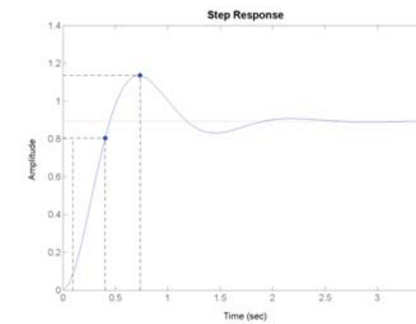
$G_c(s) = K(s+2)$

(pole-zero cancellation)



K=51.4
%OS=25.4
T_r=0.197 sec

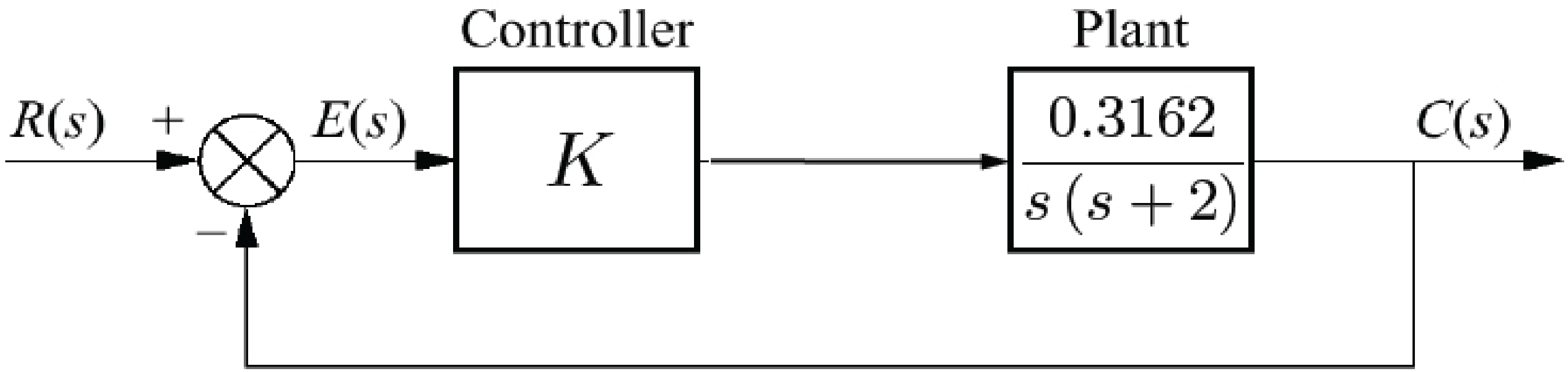


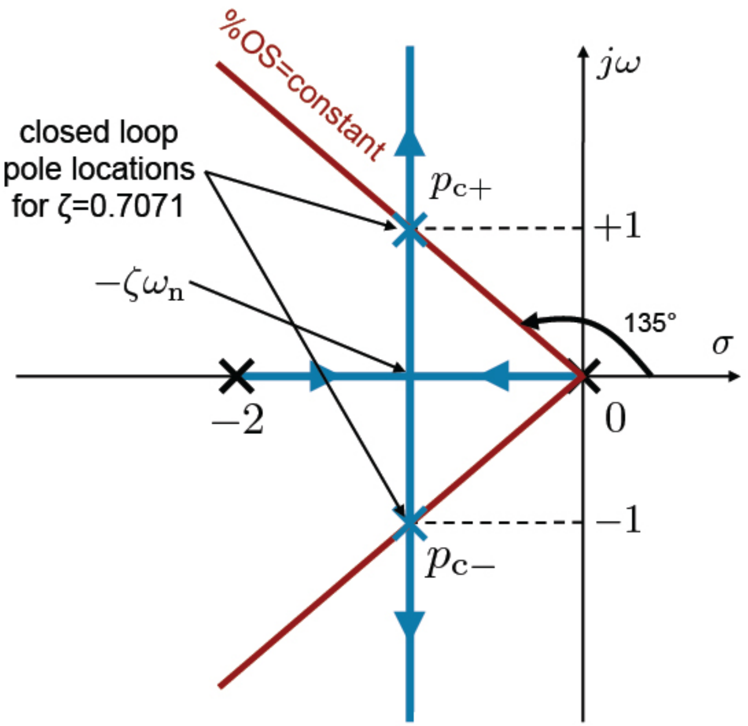


K=20.86
%OS=27.2
T_r=0.305 sec

$G_c(s) = K(s+4)$

Example: speeding up the response





The closed-loop poles shown here are with proportional control, designed for

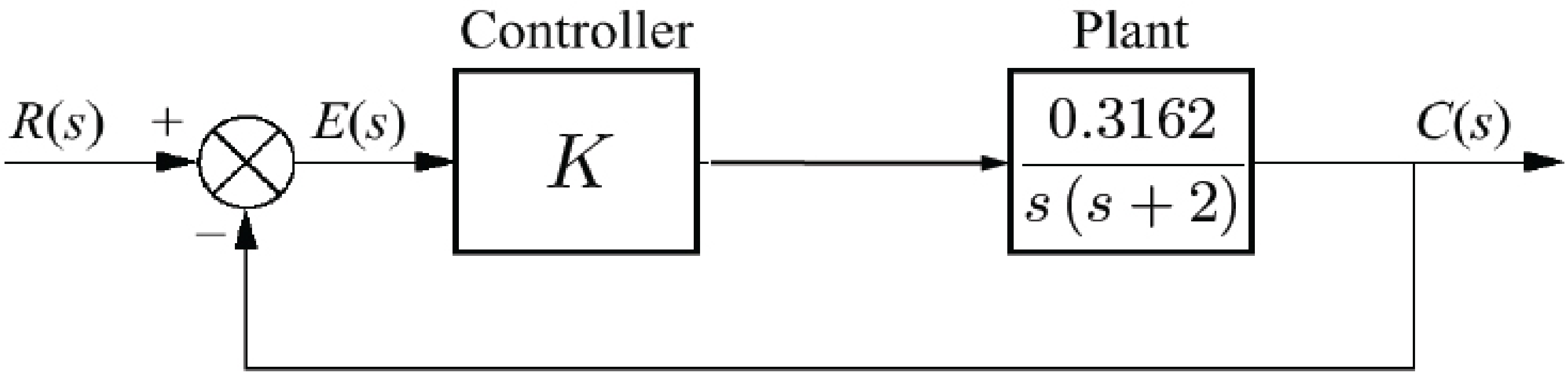
$$\zeta = 1/\sqrt{2} = 0.7071 \Leftrightarrow \%\mathrm{OS} = 4.32\%.$$

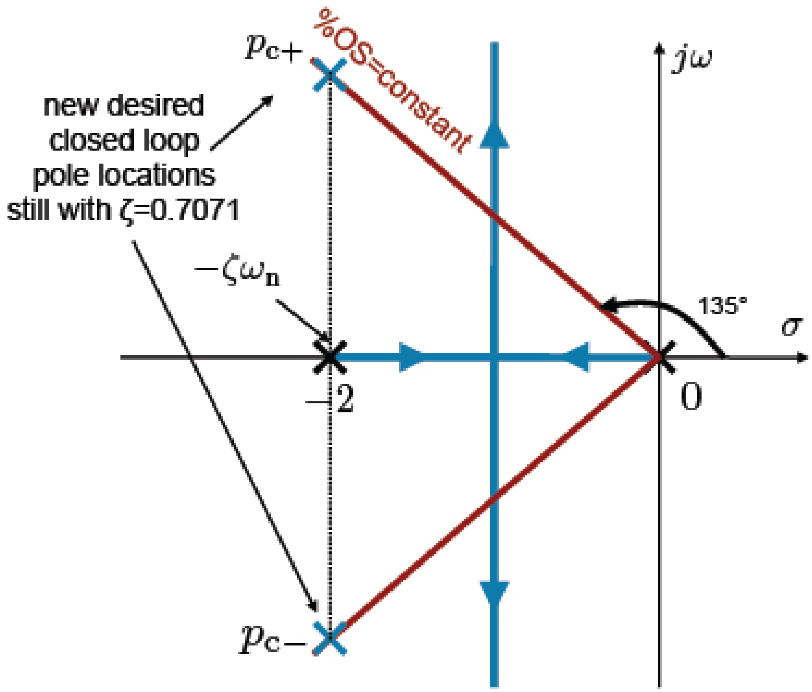
We found that the overshoot target is achieved with proportional gain $K = 6.325$. From the root locus we can see that for this value of gain, the settling time is

$$T_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{1} = 4 \text{ sec.}$$

How can we "speed up" the system to $T_s = 2$ sec while maintaining the same %OS value?

Example: speeding up the response



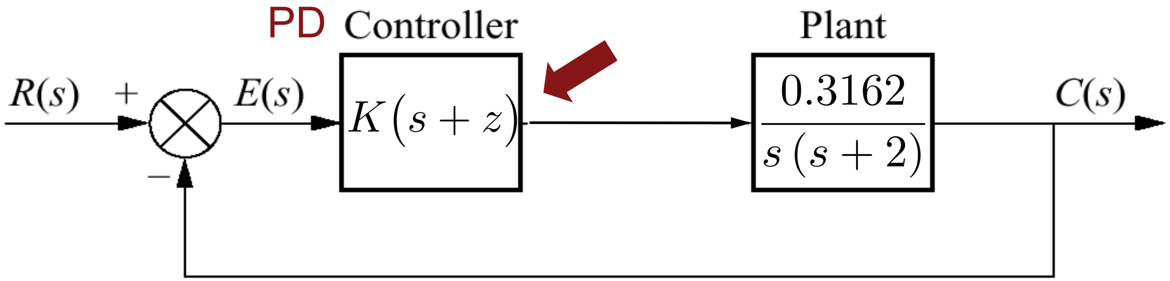


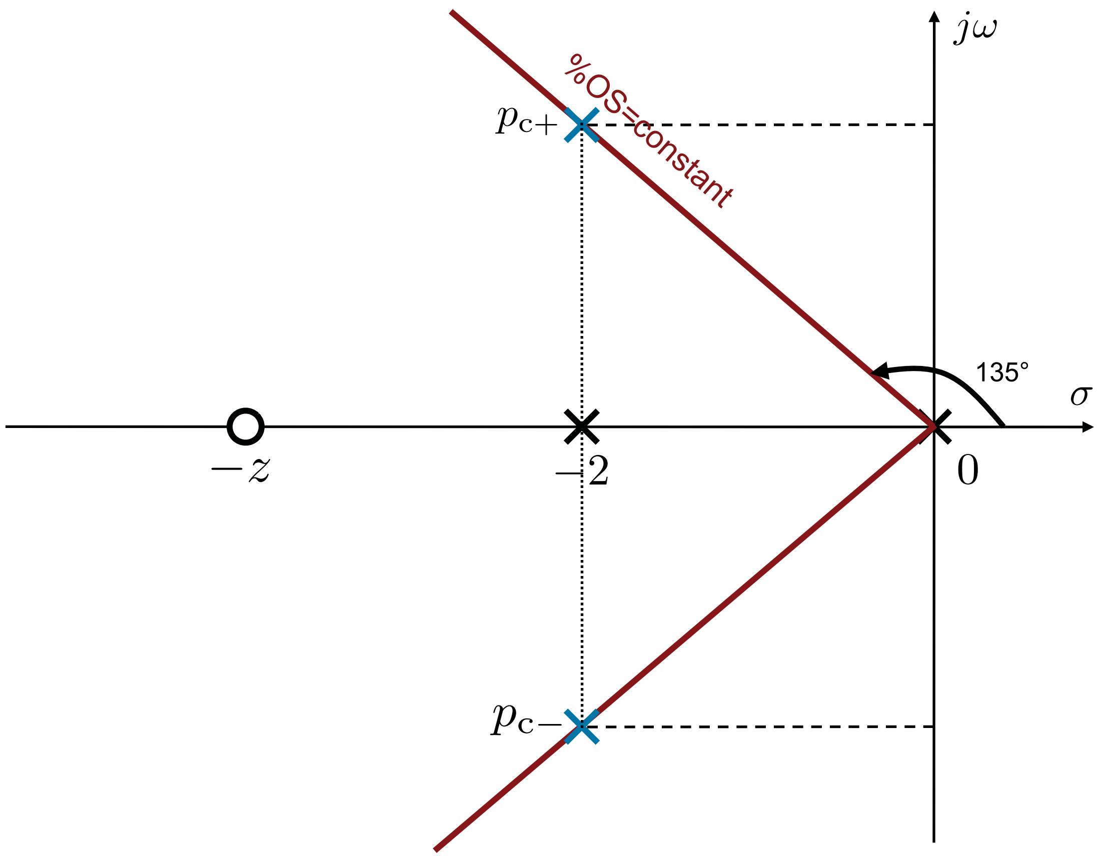
From the shorter settling time requirement, we have

$$T_s \approx \frac{4}{\zeta\omega_n} = 2 \Rightarrow \zeta\omega_n = 2.$$

Moreover, to maintain the same %OS, the poles must be located on the $\zeta = 0.707$ line. The new desired pole locations are shown on the left. Unfortunately, **they do not belong** to the uncompensated root locus. To achieve the desired poles, we propose to use a proportional–derivative (PD) compensator.

Example: speeding up the response



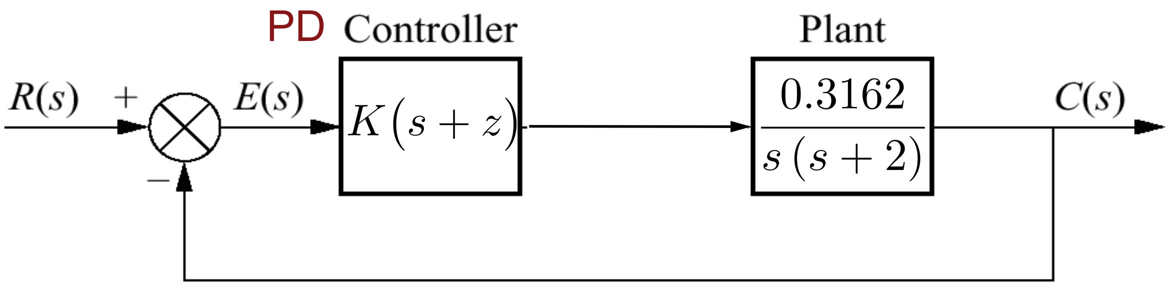


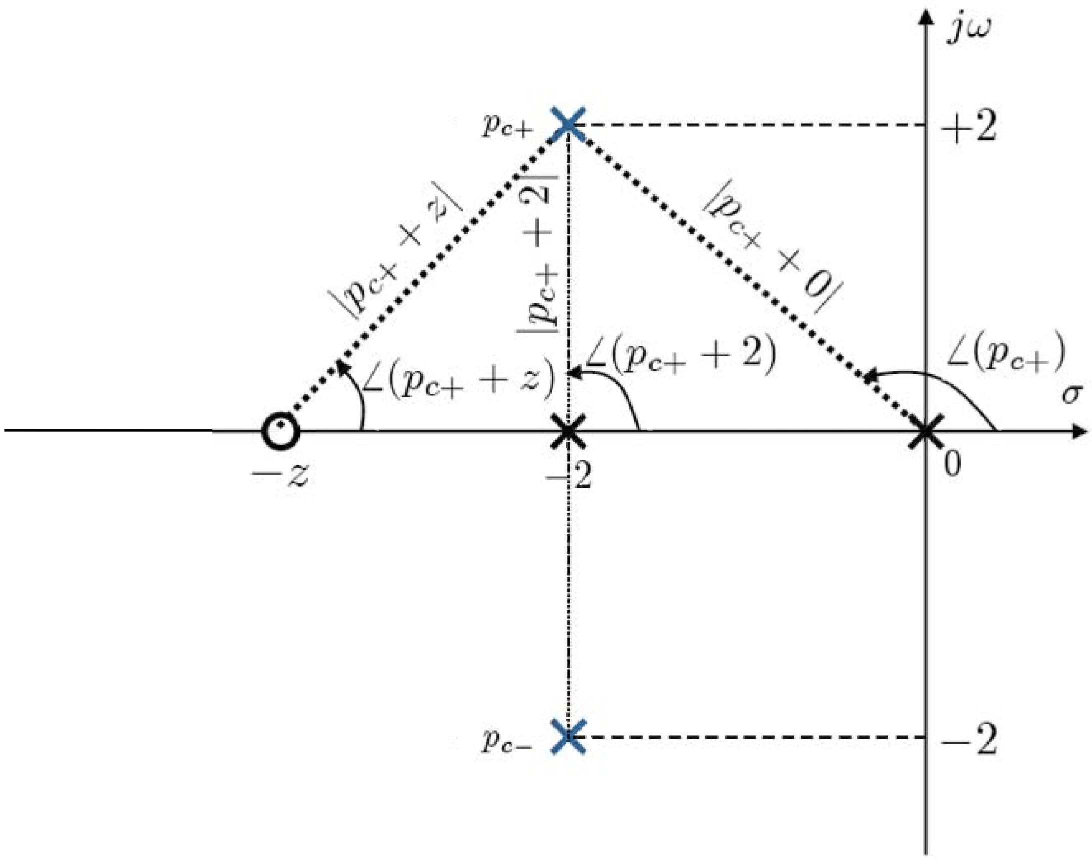
The new compensated system is represented on the left on the s–plane. The desired pole locations are

$$-p_{c\pm} = -2 \pm j2.$$

The PD compensator (see the block diagram above) contributes a zero at $-z$. We now must answer the question: where should the zero be such that the desired poles belong on the root locus? The process of placing z is the **"design of the PD compensator."**

Example: speeding up the response





If the new $p_{c\pm}$ are to belong to the root locus, the phase contributions from the open–loop poles must add up to $(2m+1)\pi$, where m is an integer.

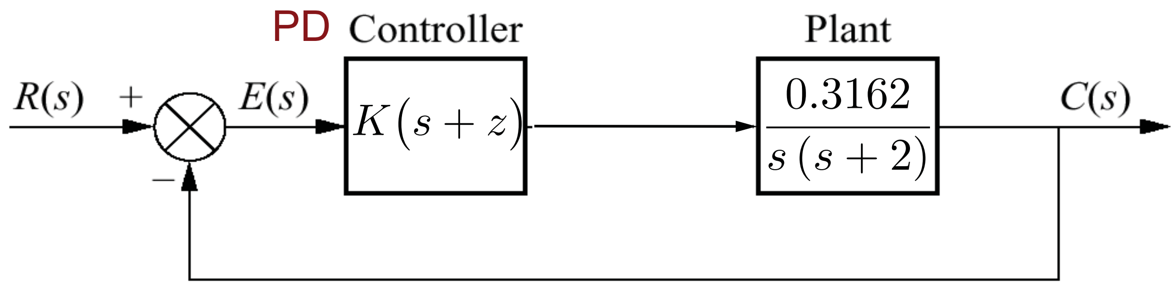
$$\angle(p_{c+} + z) - \angle(p_{c+} + 2) - -\angle(p_{c+}) = (2m+1)\pi.$$

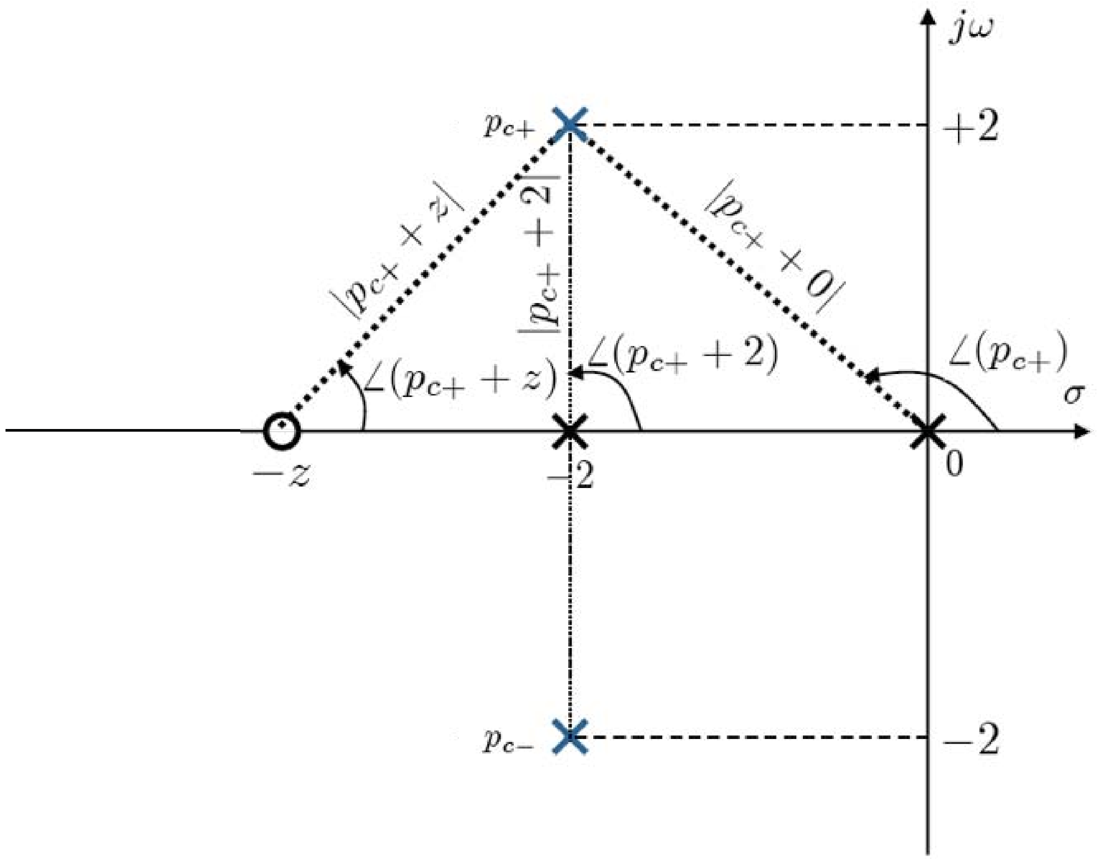
From the geometry,

$$\begin{aligned} \angle(p_{c+} + 2) &= \pi/2 \quad (90^\circ) \\ \angle(p_{c+}) &= 3\pi/4 \quad (135^\circ), \\ \Rightarrow \angle(p_{c+} + z) &= \pi/4 \quad (45^\circ), \end{aligned}$$

which places the zero at $-z = -4$.

Example: speeding up the response





Now that we have placed the zero at $-z = -4$, we can also compute the gain required to reach the desired closed–loop poles at $p_{c\pm} = 2 \pm j2$:

$$|G_c(s)|\,|G_p(s)| = 1, \quad \text{where}$$

$$G_c(s) = K(s+4)$$

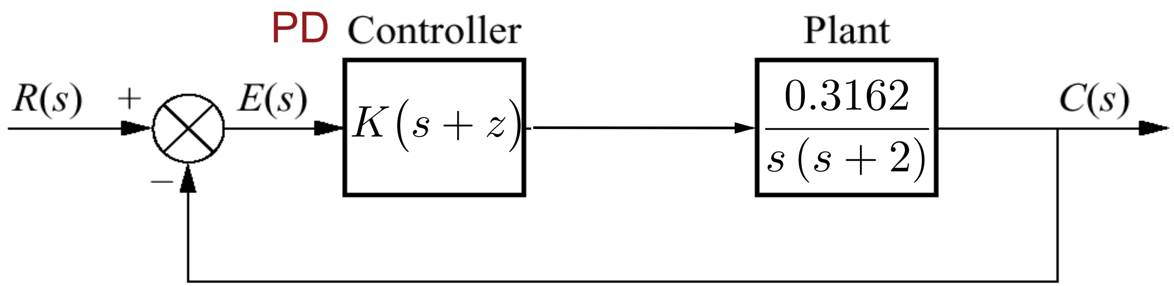
$$G_p(s) = \frac{0.3162}{s(s+2)}.$$

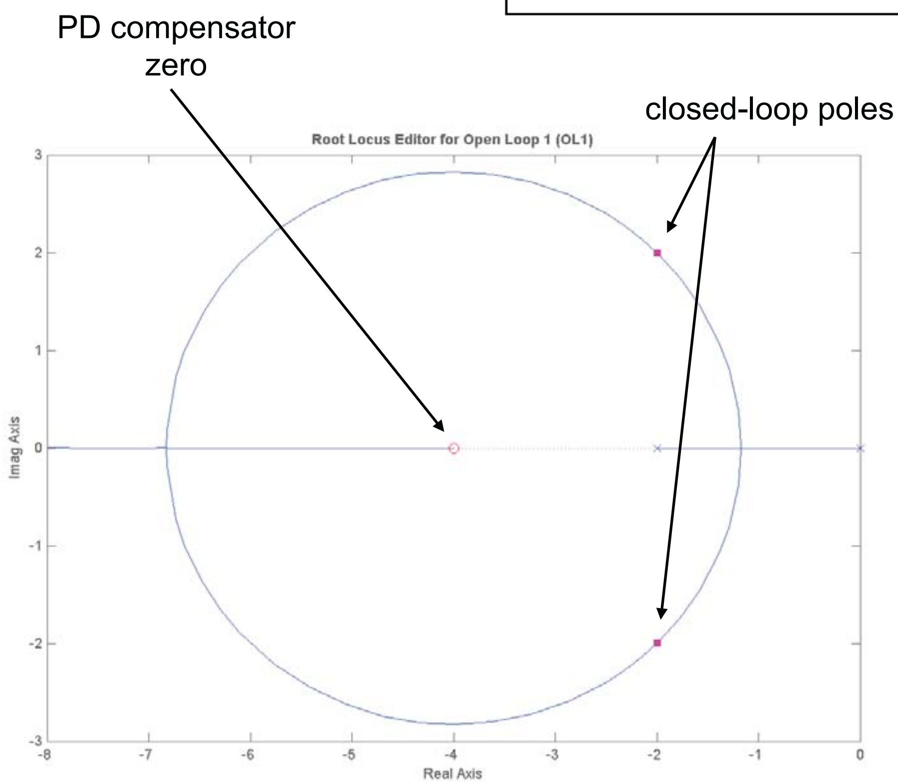
For the pole of interest at $s = -p_{c\pm}$,

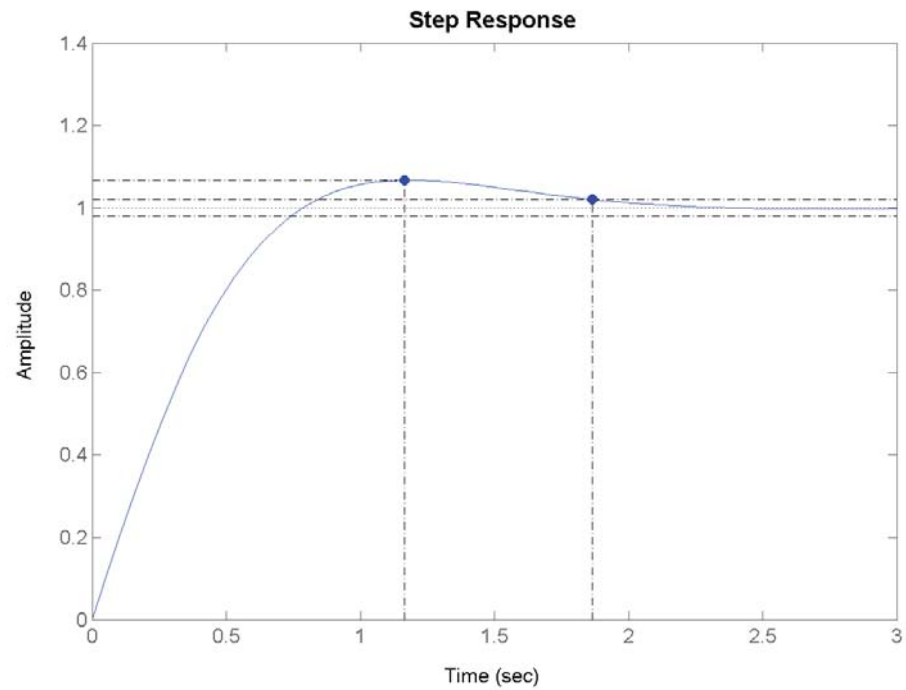
$$\Rightarrow K = \frac{|p_{c+}|\,|p_{c+} + 2|}{0.3162\,|p_{c+} + 4|}$$

$$= \frac{(2\sqrt{2}) \times 2}{0.3162 \times (2\sqrt{2})} = 6.325.$$

Example: speeding up the response







K=6.325 gives %OS=6.7; T_s=1.86sec

Experiments

- Closed-loop position control with derivative control action:
  - **Experiment #1:** P and PD Control of Position
  - **Experiment #2:** Compare your results with a Simulink Simulation

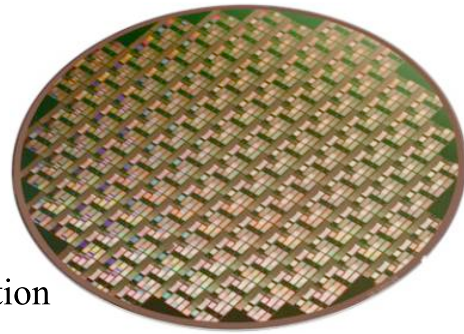
- Deliverables:
  - Properly annotated plots showing your results
  - Comments and discussions on your observations and results

Precision Position Control Example:

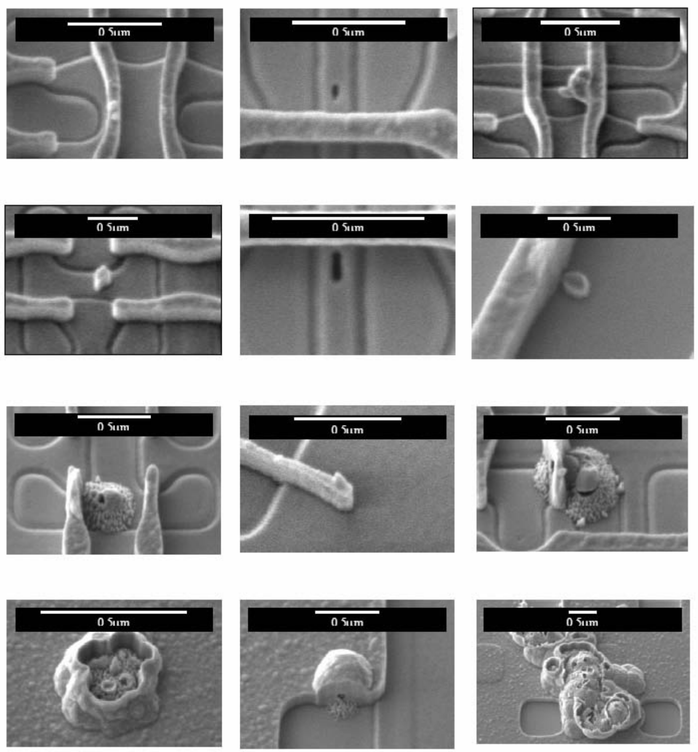
Scanning Electron Microscope (SEM) In Semiconductor Fabrication Process

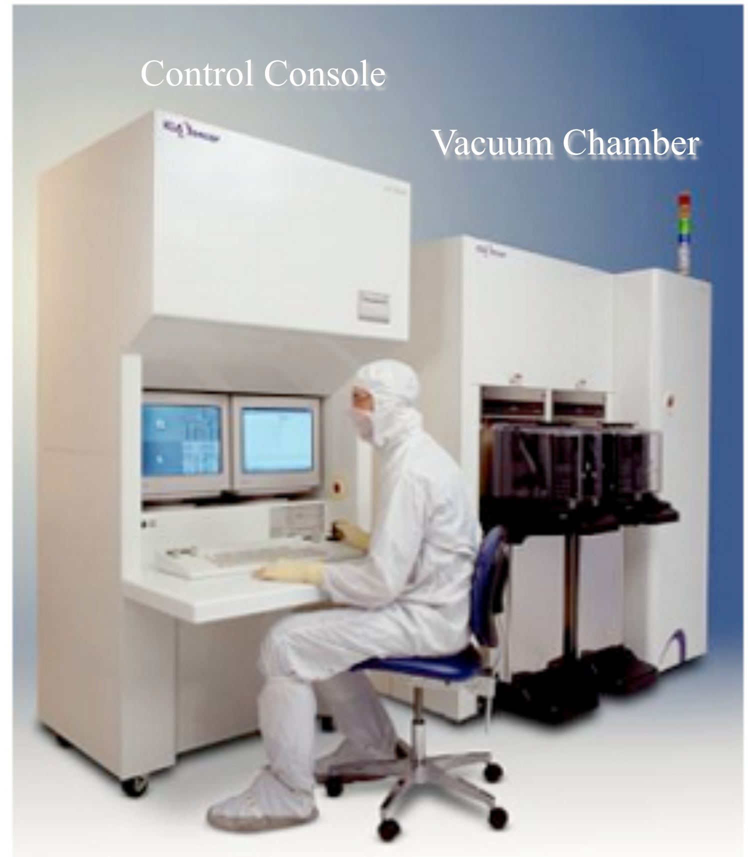
300 mm diameter wafer

45 nm feature size



200,000x magnification

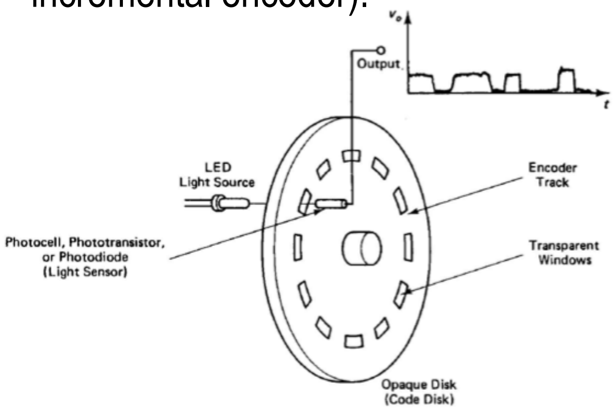


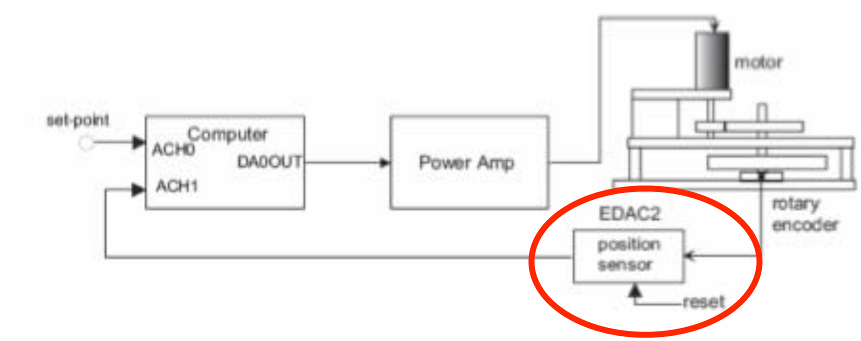


Diameter of a human hair ≈ 0.1 mm = 1e5 nm

Position Sensing Using Encoder and EDAC2

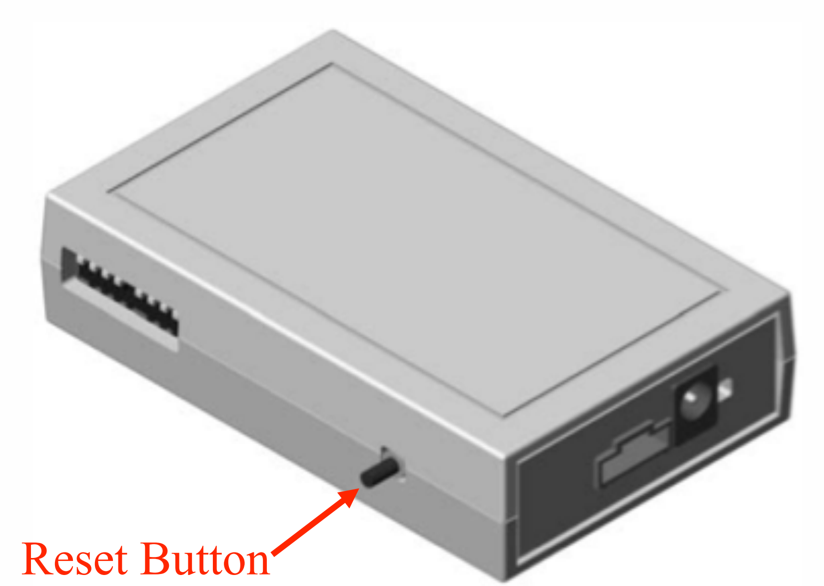
- **Encoder Types:**
  - **<u>Incremental (Relative):</u>** Only the relative position of the shaft is known. No absolute "0" position.
  - **<u>Absolute:</u>** Unique code for each shaft position (e.g., by adding a reference input to an incremental encoder).



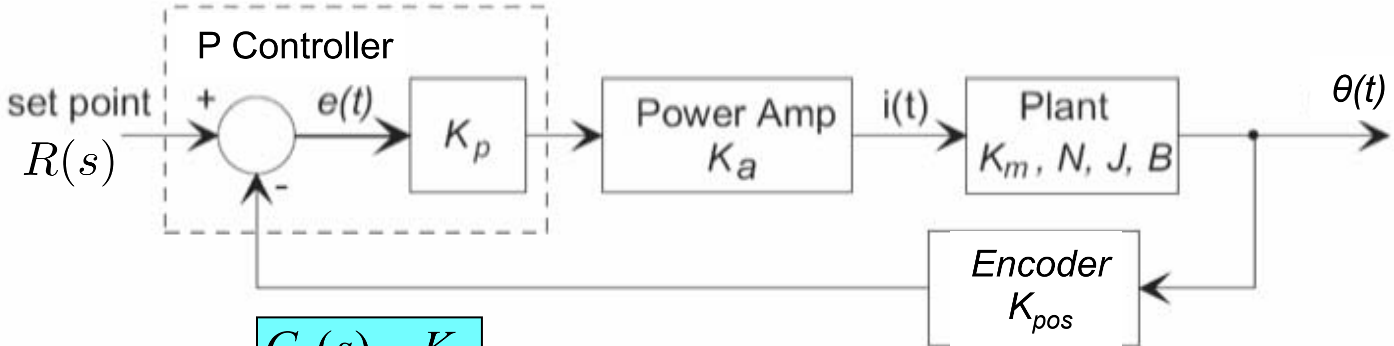


Features:

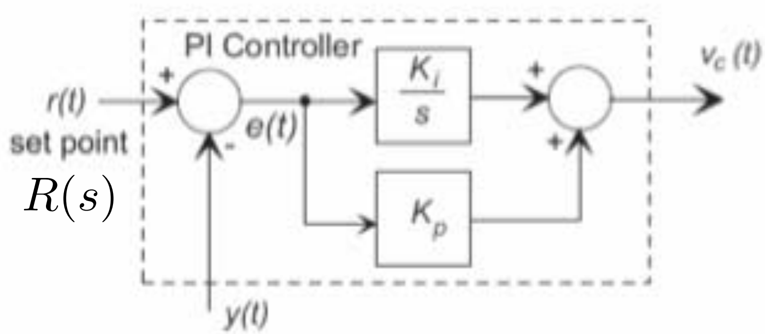
- Converts any incremental encoder into an analog position sensor
- 12 bit analog resolution
- 0 to 4.095V or 0 to 10V unipolar output voltage operation
- ±4.095 or ±10V bipolar output voltage operation
- Reset can be configured to zero or mid-range voltage
- Simple DIP switch defined programming
- DIN rail mounting is available
- TTL logic level output bit to indicate direction of rotation or linear movement
- US Digital warrants its products against defects in materials and workmanship for two years. See complete warranty for details.



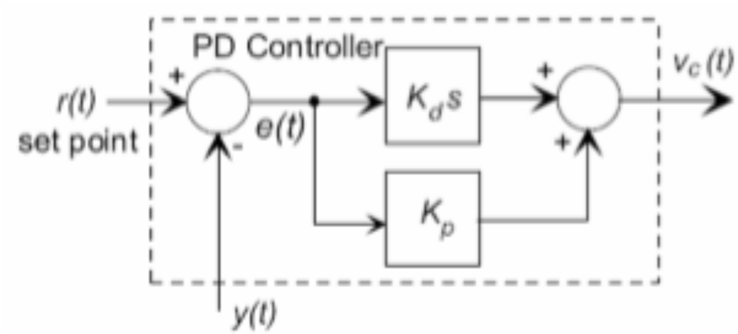
P, PI and PD Controllers



$$G_c(s) = K_p$$



$$G_c(s) = K_p + \frac{K_i}{s}$$



$$G_c(s) = K_p + K_d s$$

Plant Transfer Functions

Now we are dealing with <u>POSITION CONTROL</u>, DO NOT use the velocity transfer function.

TF voltage to velocity:

$$\frac{\Omega(s)}{V_c(s)} = \frac{K_a K_m / N}{J_{\mathrm{eq}} s + B_{\mathrm{eq}}}$$

position to velocity:

$$\frac{\Theta(s)}{\Omega(s)} = \frac{1}{s}$$

TF voltage to position:

$$G_p(s) = \frac{\Theta(s)}{V_c(s)} = \frac{1}{s} \cdot \frac{\Omega(s)}{V_c(s)} = \frac{K_a K_m / N}{s\ (J_{\mathrm{eq}} s + B_{\mathrm{eq}})}$$

Comparison of Closed-Loop Transfer Functions

MITMECHE

Let $K = K_a K_m / N$

P Control: $$G_{\mathrm{CL}}(s) = \frac{\Theta(s)}{R(s)} = \frac{KK_p}{Js^2 + Bs + KK_pK_{\mathrm{pos}}}$$

Common form $$s^2 + 2\zeta\omega_n s + \omega_n^2$$

PD Control: $$G_{\mathrm{CL}}(s) = \frac{\Theta(s)}{R(s)} = \frac{K\left(K_p + K_d s\right)}{Js^2 + \left(B + K_d K K_{\mathrm{pos}}\right)s + KK_pK_{\mathrm{pos}}}$$

System Parameters

$$J \approx 0.03 \text{ N-m}^2$$

$$B \approx 0.014 \text{ N-m-s/rad}$$

$$K_a = 2.0 \text{ A/V}$$

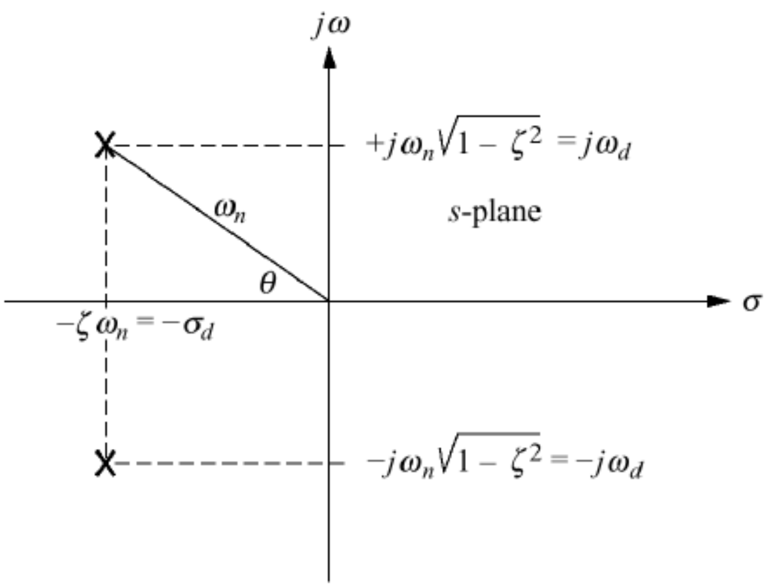
$$K_m \approx 0.0292 \text{ N-m/A}$$

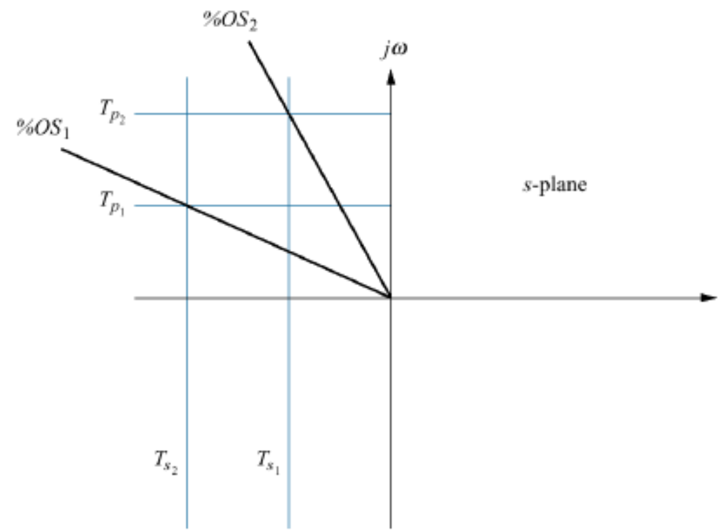
$$K_t = (0.016\frac{V}{rev/min})(60\frac{s}{min})(\frac{1rev}{2\pi rad}) = 0.153 \text{ V/(rad/s)}$$

$$N = \frac{44}{180} = 0.244$$

$$K_e \approx 1.5 \text{ V/rad}$$

2nd Order System Poles





$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\sigma_d \pm j\omega_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

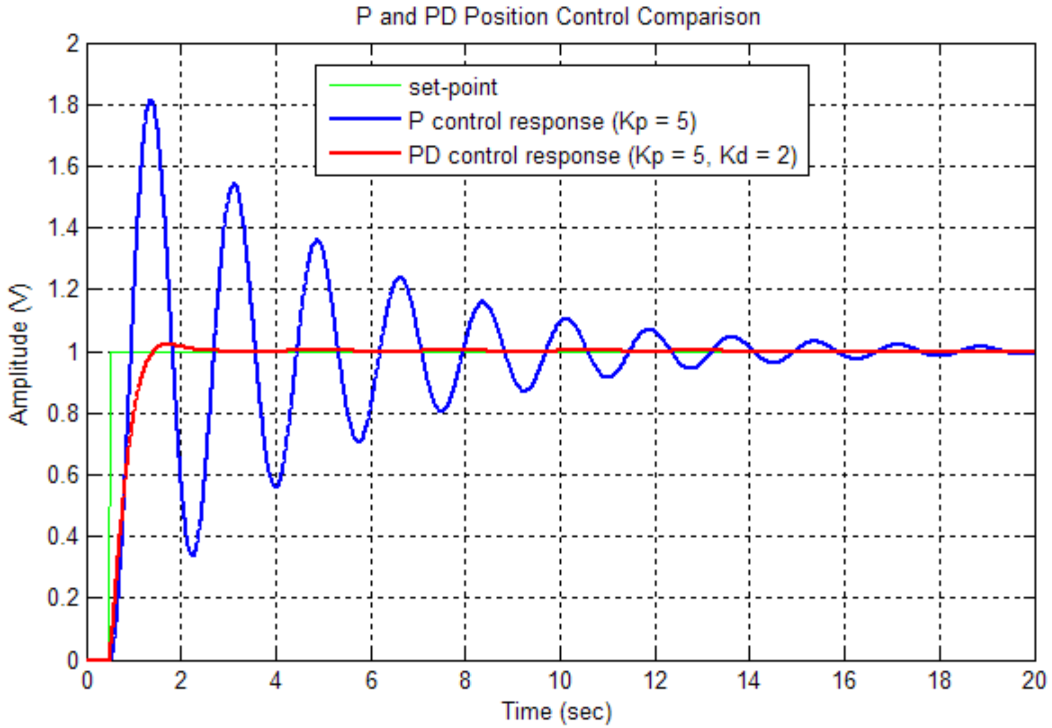
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$T_s = \frac{4}{\zeta\omega_n}$$

$$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$$

Control Action Comparison

- ***P*** **– improve speed but with steady-state error**
- ***D*** **– improve stability but sensitive to noise**
- *I – improve steady state error but with less stability, overshoot, longer transient, integrator windup (we will discuss PI and PID control next week)*



Procedure EX1

- Connect the computer-based as before except we need to use the EDAC2 instead of ETACH2. The set-up is very similar. Install one magnet.

  Important: ALWAYS RESET EDAC2 BEFORE EACH TRIAL

  (The flywheel could go out of control, be ready to stop the loop at any time.)

- Before starting the experiment, spin the flywheel so that the position mark on the flywheel is observable and define an initial position.

- Set the function generator to output a DC signal with 1.0 V offset. Set K_p to 2, 3, 4 and start experiment. Record a transient response for each case.

- Spin the wheel back to its initial position. Set K_p to 2 and K_d to 1. Repeat the experiment. Try two other (reasonable) combinations of K_p and K_d control parameters. Compare P and PD control results.

- Select your favorite combination of PD control parameters, run experiment as before after the plant reaches steady state, "continuously" change your DC offset by pressing the "up" or "down" button; observe controlling of the plant.

Procedure EXP2

- Define a Matlab SISO model to represent the flywheel with voltage as input and position as output. Simulate a P control with $K_p = 2$.
- Change your controller design to PD. Choose 2 combinations of Kp and Kd values from EXP1 and run simulation.
- Compare your simulation with results from EXP1 and comment on agreement or discrepancy between theory and experiment.