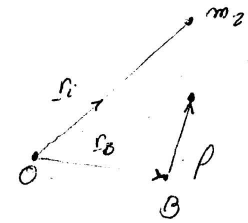
Session 5

Dynamics of systems of particles

(1) $\dot{\underline{P}} = \underline{F}^{(ext)}$

(2) $\dot{\underline{H}}_B + \underline{v}_B \times \underline{P} = \underline{M}_B$

(3) $W_{12} = T_2 - T_1$

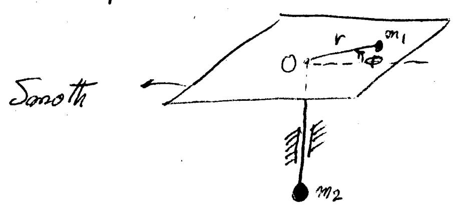


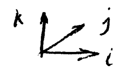
For Rigid body Systems

$W_{12}^{ext} = 0$

$T + V = Const$ for Conservative Systems

Example





$\varphi(0) = \varphi_0$

$\dot{r}(0) = 0$

$\dot{\varphi}(0) = \omega_0$

inextensible length l

Questions
- minimal r?
- maximal String force?

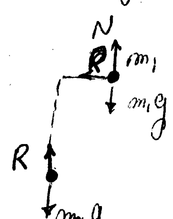
Work-energy principle $W_{12} = T_2 - T_1$

Degrees of freedom: $\#DOF = 2\times3 - 1 - 2 - 1$

Plane for m_1, String, Vertical for m_2

Use (r, ϕ) as generalized Coordinate

FBD



- N, $m_1 g$ do not work
- $m_2 g$ is potential
- work done by string forces

$$\omega_{12}^{int} = \int_1^2 (\underline{R}_1 \cdot d\underline{r}_1 + \underline{R}_2\, d\underline{r}_2) = \int_1^2 R\,dr - \int_1^2 R\,dr = 0$$

$\Rightarrow W_{12} = W_{12}^{ext}$ (all external forces are either potential or don't do work)

$= V_1 - V_2$ (V: Potential)

$\Rightarrow \underbrace{T+V}_{} = Const \Rightarrow$ System is Conservative

$$E = \frac{1}{2} m |v_1|^2 + \frac{1}{2} m_2 |v_2|^2 - mg(l - r) = Const \qquad |v_1|^2 = \dot{r}^2 + r^2\dot{\varphi}^2 \quad |v_2|^2 = \dot{r}^2$$

$$m(\dot{r}^2 + \frac{1}{2} r^2 \dot{\varphi}^2) + mgr = m(\dot{r}_0^2 + \frac{1}{2} r_0^2 \omega_0^2) + mgr_0$$

Angular momentum principle (w.r.t. o)

$$\dot{\underline{H}}_o + \cancel{\underline{v}_o \times \underline{P}}^{\,0} = \underline{M}_o = \underline{0}$$

$$\Rightarrow \underline{H}_o = \text{Const}$$

$$\underline{H}_o = \underline{r}_{om} \times \underline{P}_1 + \cancel{\underline{r}_{om_2} \times \underline{P}_2}^{\,0}$$

$$= r m \underbrace{r\dot{\varphi}}_{v_m} \qquad \Rightarrow r_0^2 \omega_0 = r^2 \dot{\varphi}$$

Combine (1) & (2): $\quad m\frac{\omega_0^2 r_0^4}{2r^2} + mgr = \frac{1}{2} m r_0^2 \omega_0^2 + mgr_0$

Cubic eq for r but we know one root at $r=r_0$ we have $\dot{r}=0$

Divide (3) by $r-r_0$ $\Rightarrow r^2 - \frac{\omega_0^2 r_0^2}{2g} r - \frac{\omega_0^2 r_0^3}{2g} = 0$

positive root: $r_{min} = \frac{\omega_0^2 r_0^2}{4g}\left(1+\sqrt{1+\frac{8g}{\omega_0^2 r_0}}\right)$

maximal force in string

linear momentum principle for m_2:

$$\dot{\underline{P}}_2 = \underline{R}_2 - m\underline{g}$$

$$\Rightarrow m\ddot{r} = R - mg \Rightarrow R = m(\ddot{r} + g) \quad (4)$$

eliminate $\dot{\varphi}$ from (1) using Conservation of Ho also set $\dot{r}=0$ at that point then $\frac{d}{dt}$ of both sides give

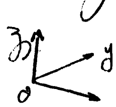
$$\left(2m\ddot{r} - m\frac{r_0^4 \omega_0^2}{r^3} + mg\right)\dot{r} = 0$$

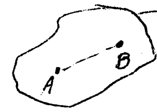
$\ddot{r} = \Rightarrow$ plug into (4) to obtain $\quad R = m\left(\frac{r_0^4 \omega_0^2}{2r^3} + \frac{g}{2}\right)$

R_{max} occures at r_{min} $\quad R_{max} = m\left(\frac{r_0^4 \omega_0^2}{2 r_{min}^3} + \frac{g}{2}\right)$

III Dynamics of Rigid Bodies

Rigid body

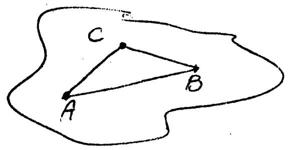




→ Continuum of particles

$|\underline{r}_A - \underline{r}_B| = \text{Const}$

for all A & B on the body



$\# DOF = 3 \times 3 - \underset{\text{Constraint}}{3} = 6$

$|\underline{r}_A - \underline{r}_B| = Const$

$|\underline{r}_B - \underline{r}_C| = Const$

$|\underline{r}_A - \underline{r}_C| = Const$

General motion of a rigid body

Can always be viewed as a superposition of translation and a rotation about a fixed ~~axis~~ Point