Session 13

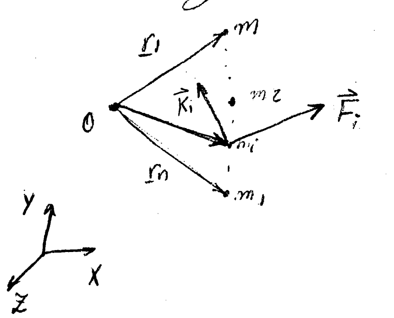
Variational Aproach to dynamics

(Lagrangian mechanics, energy Principle)

- Scalar (work-energy-based) (as opposed to vector based)
- frame invariant
- renders equations of motion without (in most cases) getting the Constraint reaction forces involved
- Reaction forces are not immediately available (disadvantage)

INGRADIANTS

i) Generalized Coordinates



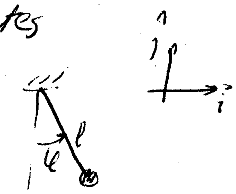
$\vec{F}_i$: active force
$\vec{K}_i$: Constraint force

Often it is possible to select a smaller set of Coordinate (not necessarily position) that uniquely determine the position of the system and already account for (some of the) Construction Paints

Eg. q_i : generalized Coordinates

$x_1 = x_1(q_1, \ldots, q_N)$
$y_1 = y_1(- \quad -)$
$z_1 = z_1(- \quad -)$

$y = l\cos\theta$
$x = l\sin\theta$



NOTE q_i is a Complete Set of Coordinates, but not necessarily independent

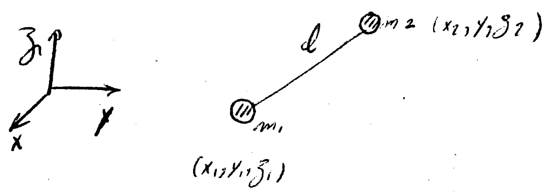
ii) Constraints Scalar relations that limit possible motions of the System

$f_j(r_i, \dot{r}_i, t) = 0 \qquad j = 1, \ldots, m$

types of Constraints

types of Constraint	scleronomic	rheonomic (time dependent)
holonomic	$f(r_i) = 0$	$f(r_i, t) = 0$
non-holonomic	$f(r_i, \dot{r}_i) = 0$	$f(r_i, \dot{r}_i, t)$

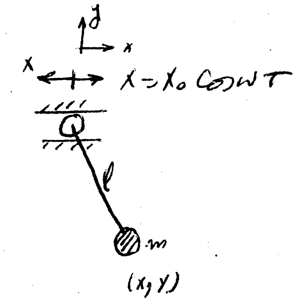
Example (1) Moving ~~[illegible]~~ dumbrell



$f = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - l^2 = 0$

holonomic scleronomic Constraint

(2) Pendulum with Oscillating Support



$(X - x_0 \cos \omega t)^2 + y^2 - l^2 = 0$

holonomic Rheonomic Constraint

II Analytical Mechanics

Ingredients (1) generalized Coordinates $r_i = r_i(q_1, \dots, q_n, t)$ $i = 1, \dots, n$

Complete (not necessarily independent)

(2) Constraints $f_j(q, \dot{q}, t) = 0$, $j = 1, \dots, m$

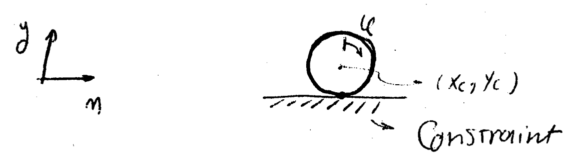
holonomic
non-holonomic

$q = \begin{Bmatrix} q_1 \\ \vdots \\ q_n \end{Bmatrix}$

scleronomic
rheonomic

#DOF = unConstraint #DOF − m
(3n)

Example 3, 2D rolling



#DOF = 3 − 2 = 1

$\underline{V}_P = \underline{0} \Rightarrow \underline{V}_P = \underline{V}_C + \underline{\omega} \times \underline{r}_{CP}$

$\Rightarrow \left.\begin{matrix} \dot{x} - R\dot{\varphi} = 0 \\ \dot{y} = 0 \end{matrix}\right\}$ $(m = 2)$

Strictly Speaking (*) gives 2 nonholonomic Scleronomic Constraints
But $\int dt$ gives

$\left.\begin{matrix} (x_c - R\varphi) - (x_c^0 - (R\varphi^0)) = 0 \\ y_c = y_{c_0} = R \end{matrix}\right\}$ integrated nonholonomic Constraint (Semi holonomic)

$\begin{cases} x_c = x_c(\varphi) \\ y_c = y_c(\varphi) = R \end{cases}$

a Complete and independent Set of generalized Coordinate is $\{\varphi\}$