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Session 10

$$\Rightarrow \quad L = \frac{1}{6} M L^2 \dot\varphi^2 + \frac{1}{2} m \left[\dot r^2 + r^2 \dot\varphi^2\right] + g \left(\frac{ML}{2} + mr\right) \cos\varphi$$

= Recall from Virtual work $\Rightarrow$ $Q_\varphi^{\text{non-potential}} = F\ell \sin\alpha$

$$Q_r^{\text{non-pot}} = \mu m \left| r\ddot\varphi + 2\dot r\dot\varphi + g \sin\varphi \right| \cdot \text{Sign}(\dot r)$$

$\Rightarrow$ Equation of motion

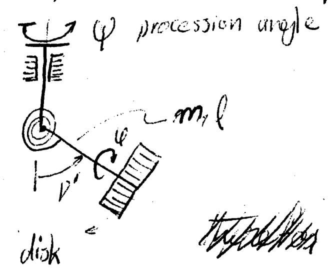
$$\frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} - \frac{\partial L}{\partial \varphi} = Q_\varphi \Rightarrow \left(\frac{1}{3} M L^2 + m r^2\right)\ddot\varphi + 2 m r \dot r \dot\varphi + g\left(\frac{ML}{2} + mr\right)\sin\varphi = F\ell \sin\alpha$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot r} - \frac{\partial L}{\partial r} = Q_r \Rightarrow m\ddot r - m r \dot\varphi^2 - m g \cos\varphi = -\mu m \left| r\ddot\varphi + 2\dot r\dot\varphi + g \sin\varphi \right| \cdot \text{Sign}(\dot r)$$

EXAMPLE illustrates frame independente of Lagrangian approach

$g \downarrow$

Torsional Spring (k)



Assume: Spring is unstretched for $\nu = 0$

DOF: $2 \times 6 - 5 - 4 = 3$

System is holonomic

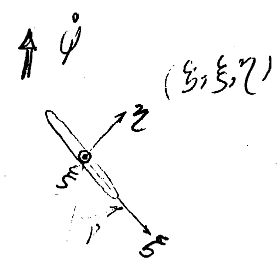
Lagrangian equation of motion applies

apply $L(\varphi, \nu, \psi, \dot\varphi, \dot\nu, \dot\psi)$

- Forces – Constraint forces are ideal
- Active forces are potential $\rightarrow \underline{Q}_q = \underline{0}$

$$T = T^{\text{beam}} + T^{\text{disk}} \; ; \; V = V^{\text{beam}} + V^{\text{disk}} + V^{\text{spring}}$$

$$T^{\text{beam}} = \frac{1}{2} m |V_c|^2 + \frac{1}{2} \underline{\omega}^{\text{beam}} \underline{\underline{I}}_c \, \underline{\omega}^{\text{beam}}$$



Consider a frame which is principal fix for beam.

$$\Rightarrow T_{beam} = \frac{1}{2} m \left[\left(\frac{L}{2}\sin\nu \, \dot\varphi\right)^2 + \left(\frac{L}{2}\dot\nu\right)^2\right]$$
$$+ \frac{1}{2}\left[I^b_{\xi\xi}(-\dot\varphi \cos\nu)^2 + I^b_{\zeta\zeta}(-\dot\nu)^2 + I^b_{\eta\eta}(\dot\varphi \sin\nu)^2 \right]$$

$$I^b_{\xi\xi} = 0 \text{ (Slender beam)} \quad I^b_{\eta\eta} = I^b_{\zeta\zeta} = \frac{1}{12} m L^2$$

$$T^{\text{beam}} = \frac{1}{6} m L^2 \left(\sin^2\nu \, \dot\varphi^2 + \dot\nu^2\right)$$

$$T^{\text{disk}} = \frac{1}{2} M |V_0|^2 + \frac{1}{2} \left(\underline{\omega}^{\text{disk}}\right)^T I_c \, \underline{\omega}^{\text{disk}}$$

Use previous frame shifted to B

$\Rightarrow T_{disk} = \frac{1}{2} M \left[(L \sin\nu \dot{\varphi})^2 + (L\dot{\nu})^2 \right]$

$+ \frac{1}{2} \left[I^d_{\xi\xi} (-\dot{\varphi} - \dot{\psi} \cos\nu) + I^d_{\xi\xi} (-\dot{\nu})^2 + I^d_{\eta\eta} (\dot{\psi} \sin\nu)^2 \right]$

Note: $I^d_{\xi\xi} = \frac{1}{2} MR^2$, $I^d_{\xi\xi} = I^d_{\eta\eta} = \frac{1}{4} MR^2$

$\Rightarrow T^{disk} = \frac{1}{2} M \left[L^2 + \frac{1}{4} R^2 \right] \sin^2\nu \dot{\psi}^2 + \frac{1}{2} M (L^2 + R^2) \dot{\nu}^2 + \frac{1}{4} MR^2 (\dot{\varphi} + \dot{\psi} \cos\nu)^2$

$V = -mg\frac{L}{2} \cos\nu - MgL \cos\nu + \frac{1}{2} K \nu^2$

$L = T - V = \ldots$

$\Rightarrow$ Eq of motion $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0$

$q = \psi, \nu, \varphi$

φ, ψ are called cyclic Coordinate (ignorable)

Symmetry

$\downarrow$

Lagrangian doesn't depend on them explicitly

Conservation of angular momentum $\leftarrow$ $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\psi}} \right) - \frac{\partial L}{\partial \psi} = 0$, $\frac{\partial L}{\partial \psi} \to 0$

$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\nu}} \right) - \frac{\partial L}{\partial \nu} = 0$

$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\varphi}} \right) - \frac{\partial L}{\partial \varphi} = 0$