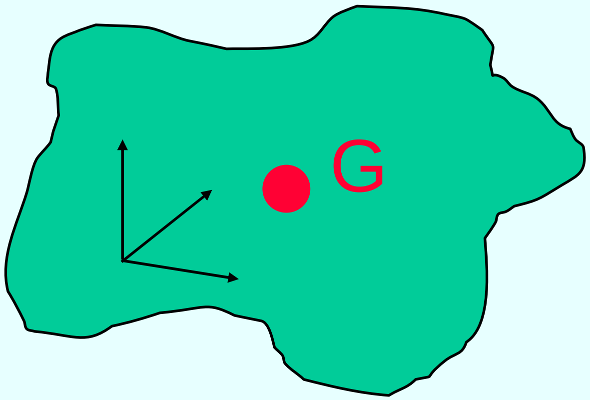
STABILITY AND TRIM OF MARINE VESSELS

Acknowledgements to
Lt. Greg Mitchell for Slides 15-37

Concept of Mass Center for a Rigid Body



Centroid – the point about which moments due to gravity are zero:

$\Sigma\, g\, m_i\, (x_g - x_i) = 0 \rightarrow$

$x_g = \Sigma\, m_i\, x_i \,/\, \Sigma\, m_i = \Sigma\, m_i\, x_i \,/\, M$

- Calculation applies to all three body axes: x,y,z
- x can be referenced to any point, e.g., bow, waterline, geometric center, etc.
- "Enclosed" water has to be included in the mass if we are talking about inertia

Center of Buoyancy

A similar differential approach with *displaced mass*:

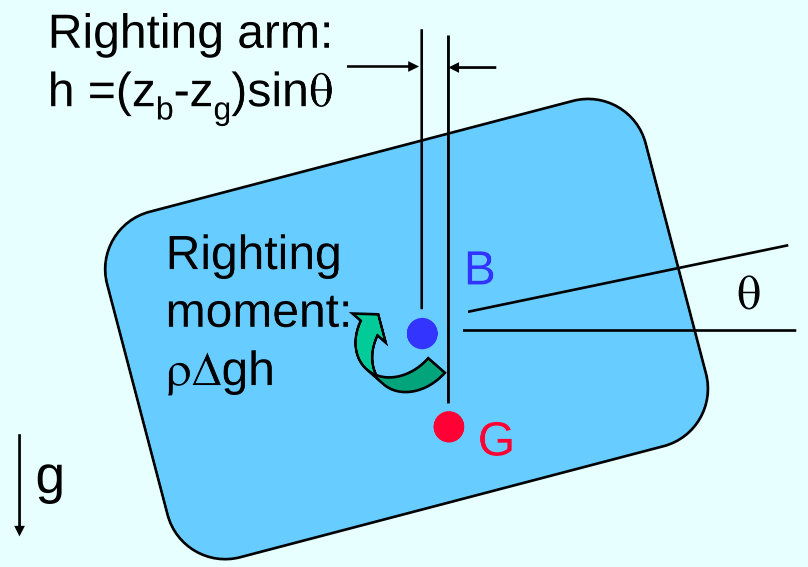
$\mathbf{x_b = \Sigma \Delta_i x_i} / \Delta$, where Δ_i is incremental volume,

Δ is total volume

Center of buoyancy is the <u>same</u> as the center of displaced volume: it doesn't matter what is inside the outer skin, or how it is arranged.

Calculating trim of a flooded vehicle: Use <u>in-water</u> weights of the components, including the water (whose weight is then zero and can be ignored). The calculation gives the center of in-water weight.

- For a <u>submerged body</u>, a sufficient condition for stability is that z_b *is above* z_g.

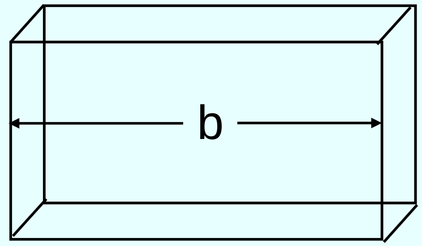


Make $(z_b - z_g)$ large → the "spring" is large and:

- Response to an initial heel angle is fast (uncomfortable?)
- Wave or loading disturbances don't cause unacceptably large motions
- But this is also a spring-mass system, that will oscillate unless adequate damping is used, e.g., sails, anti-roll planes, etc.

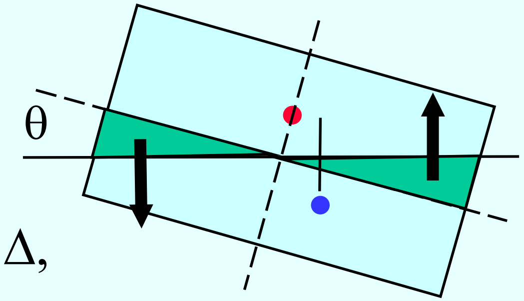
- In most <u>surface vessels</u>, righting stability is provided by the *waterplane area*.

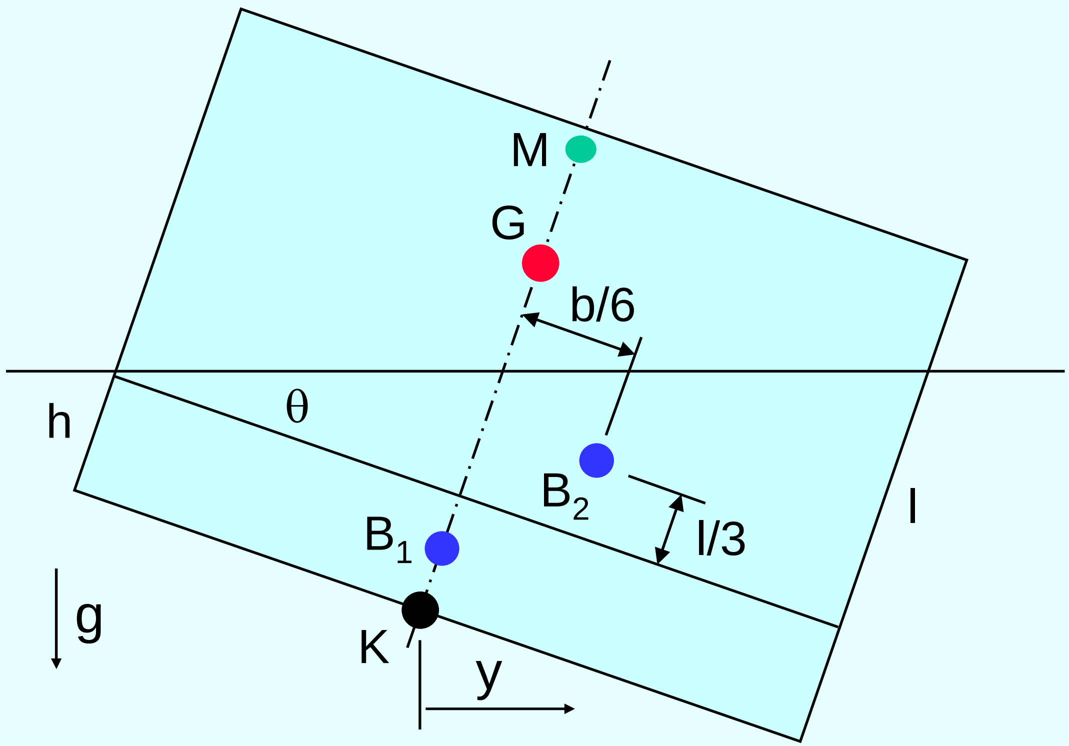
slice thickness dx



submerged volume dΔ,
$d\Delta = A dx$

<u>intuition</u>: wedges





RECTANGULAR SECTION

Geometry:

$d\Delta/dx = bh + bl/2$ or

$h = (d\Delta/dx - bl/2) / b$

$l = b \tan\theta$

Vertical forces:

$dF_G = -\rho g d\Delta$ (no shear)

$dF_{B1} = \rho g b h dx$

$dF_{B2} = \rho g b l dx / 2$

Moment arms:

$y_G = KG \sin\theta$; $y_{B1} = h \sin\theta / 2$; $y_{B2} = (h + l/3) \sin\theta + b \cos\theta / 6$

Put all this together into a net moment (positive anti-clockwise):

$$dM/\rho g = -KG\, d\Delta \sin\theta + bh^2\, dx \sin\theta / 2 + b\, l\, dx\, [\,(h+l/3) \sin\theta + b \cos\theta / 6]\, / 2$$

(valid until the corner comes out of the water)

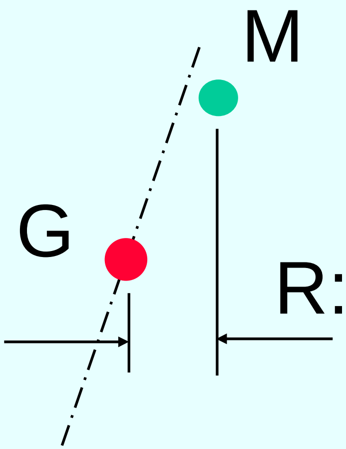
Linearize ($\sin\theta \sim \tan\theta \sim \theta$), and keep only first-order terms (θ):

$$dM / \rho g\, d\Delta = [-KG + h/2 + b^2/12h]\ \theta$$
$$= [-KG + A/2b + b^3/12A]\ \theta$$

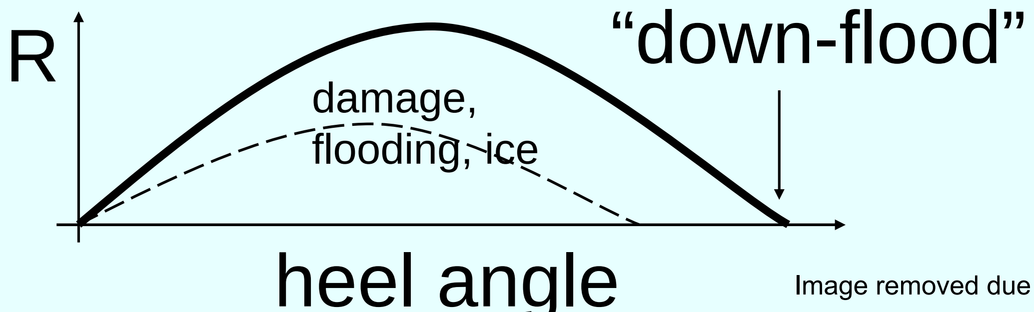
For this rectangular slice, *the sum [h / 2 + b^2 / 12 h] must exceed the distance KG for stability*. This sum is called KM – the distance from the keel up to the "virtual" buoyancy center M. M is the METACENTER, and it is as if the block is hanging from M!

-KG + KM = GM : the METACENTRIC HEIGHT

*Note M does not have to stay on the centerline**!*



R: righting arm



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such as http://www.imdb.com/media/rm1041734656/tt0177971

How much GM is enough?

Around 2-3m in a big boat

The Perfect Storm

Considering the Entire Vessel…

Transverse (or roll) stability is calculated using the same moment calculation extended on the length:

Total Moment = Integral on Length of dM(x), where (for a vessel with all rectangular cross-sections)

$$\mathbf{dM(x) = \rho g\,[\,-KG(x)\,A(x) + A^2(x)\,/\,2\,b(x) + b^3(x)\,/\,12\,]\,dx\,\theta}$$

First term: Same as $-\rho$ g KG Δ, if Δ is the ship's submerged volume, and KG is the value referencing the whole vessel

Second term: Significant if d>b (equivalent to h^2 b / 2)

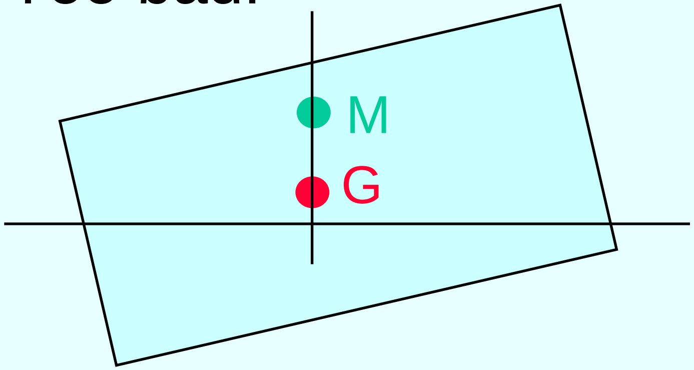
Third term: depends only on beam – dominant for most monohulls

Longitudinal (or pitch) stability is similarly calculated, but it is usually secondary, since the waterplane area is very long → very high GM

Weight Distribution and Trim

- At zero speed, and with no other forces or moments, the vessel has B (submerged) or M (surface) *directly above* G.

Too bad!



For port-stbd symmetric hulls, keep G on the centerline using a tabulation of component masses and their centroid locations in the hull, i.e., $\Sigma m_i y_i = 0$

Longitudinal trim should be zero relative to center of <u>waterplane area</u>, in the loaded condition.

Pitch trim may be affected by forward motion, but difference is usually only a few degrees.

Rotational Dynamics Using the Centroid

Equivalent to $F = ma$ in linear case is

$$\mathbf{T = J_o * d^2\theta / dt^2}$$

where T is the sum of acting torques in roll
J_o is the rotary moment of inertia in roll, referenced to some location O
θ is roll angle (radians)

J written in terms of <u>incremental</u> masses m_i:

$$J_o = \Sigma m_i (y_i - y_o)^2 \quad \text{OR} \quad J_g = \Sigma m_i (y_i - y_g)^2$$

J written in terms of <u>component</u> masses m_i and their own moments of inertia J_i (by the parallel axis theorem) :

$$J_g = \Sigma m_i (y_i - y_g)^2 + \Sigma J_i$$

The y_i's give position of the centroid of each body, and J_i's are referenced to those centroids

What are the acting torques T ?

- Buoyancy righting moment – metacentric height
- Dynamic loads on the vessel – e.g., waves, wind, movement of components, sloshing
- Damping due to keel, roll dampers, etc.
- Torques due to roll control actuators

An instructive case of damping D, metacentric height GM:

$$J\, d^2\theta / dt^2 = - D\, d\theta/dt - GM\, \rho\, g\, \Delta\, \theta \quad OR$$

$$J\, d^2\theta / dt^2 + D\, d\theta/dt + GM\, \rho\, g\, \Delta\, \theta = 0$$

$$d^2\theta / dt^2 + a\, d\theta/dt + b\theta = 0$$

$$d^2\theta / dt^2 + 2\, \zeta\, \omega_n\, d\theta/dt + \omega_n^2\theta = 0$$

A second-order stable system → Overdamped or oscillatory response from initial conditions

Homogeneous Underdamped Second-Order Systems

$x'' + ax' + bx = 0$; *write as* $x'' + 2\zeta\omega_n x' + \omega_n^2 x = 0$

Let $x = X e^{st}$ ➔
$(s^2 + 2\zeta\omega_n s + \omega_n^2)\, e^{st} = 0$ OR $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ ➔
$s = [-2\zeta\omega_n \,+/-\, \mathrm{sqrt}(4\zeta^2\omega_n^2 - 4\omega_n^2)] / 2$
$= \omega_n[-\zeta \,+/-\, \mathrm{sqrt}(\zeta^2-1)]$ *from quadratic equation*

s_1 and s_2 are complex conjugates if $\zeta < 1$, in this case:
$s_1 = -\omega_n\zeta + i\omega_d$, $s_2 = -\omega_n\zeta - i\omega_d$ where $\omega_d = \omega_n\, \mathrm{sqrt}(1-\zeta^2)$

Recalling $e^{r+i\theta} = e^r (\cos\theta + i \sin\theta)$, we have
$x = e^{-\zeta\omega n t} [(X_1^r + iX_1^i)(\cos\omega_d t + i\sin\omega_d t) +$
$(X_2^r + iX_2^i)(\cos\omega_d t - i\sin\omega_d t)]$ AND

$x' = -\zeta\omega_n x + \omega_d e^{-\zeta\omega n t}$ [$(X_1^r + iX_1^i)(-\sin\omega_d t + i\cos\omega_d t)$ +
$(X_2^r + iX_2^i)(-\sin\omega_d t - i\cos\omega_d t)$]

Consider initial conditions $x'(0) = 0$, $x(0) = 1$:

$x(t=0) = 1$ means $X_1^r + X_2^r = 1$ (real part) and
$X_1^i + X_2^i = 0$ (imaginary part)

$x'(t=0) = 0$ means $X_1^r - X_2^r = 0$ (imaginary part) and
$-\zeta\omega_n + \omega_d(X_2^i - X_1^i) = 0$ (real part)

Combine these and we find that

$X_1^r = X_2^r = \frac{1}{2}$

$X_1^i = -X_2^i = -\zeta\,\omega_n / 2\,\omega_d$

Plug into the solution for x and do some trig:

$\mathbf{x = e^{-\zeta\omega n t} \sin(\omega_d t + k) / sqrt(1-\zeta^2)}$, where $\mathbf{k = atan(\omega_d/\zeta\omega_n)}$

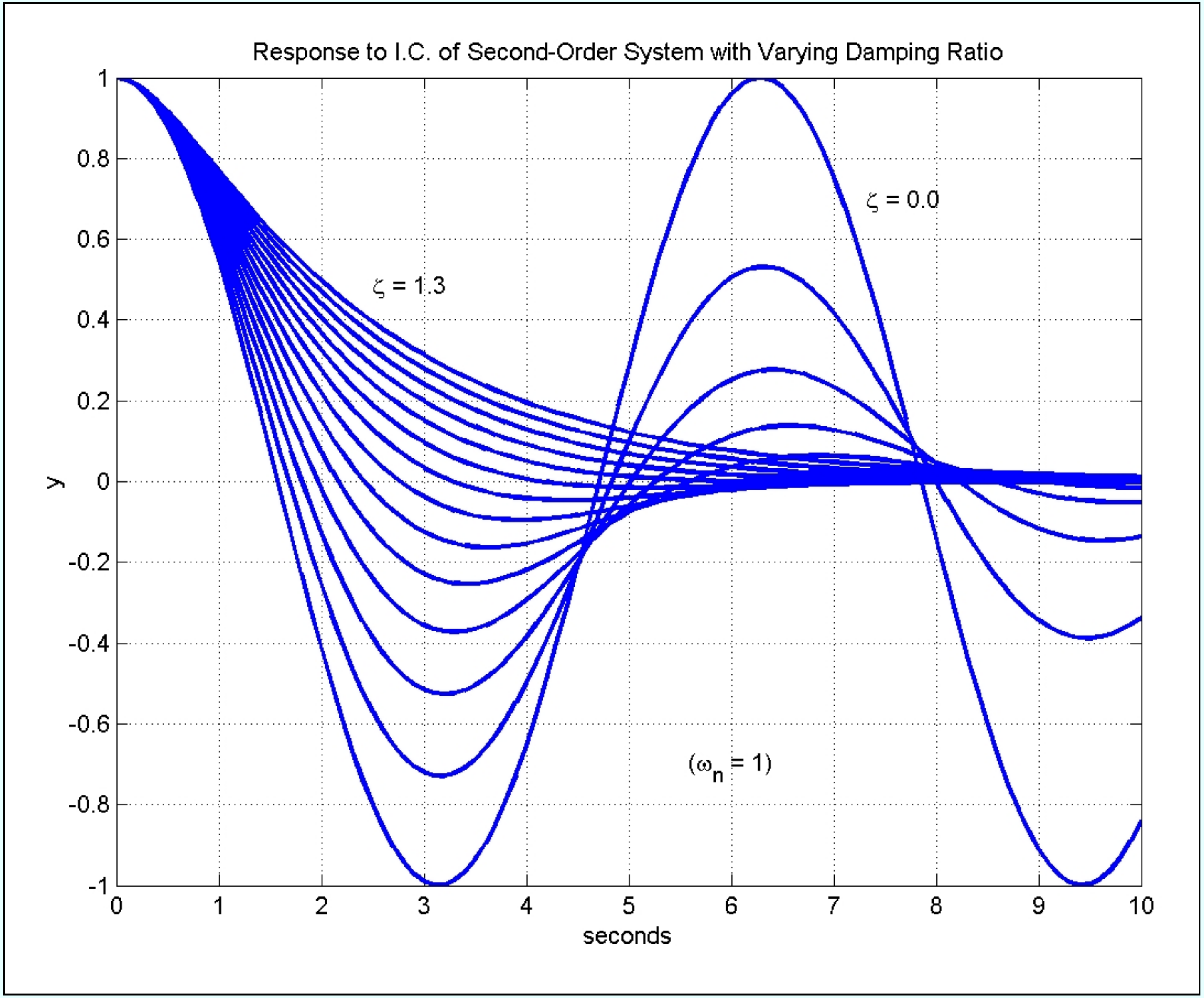
ζ= 0.0 has fastest rise time but no decay

ζ = 0.2 gives about 50% overshoot

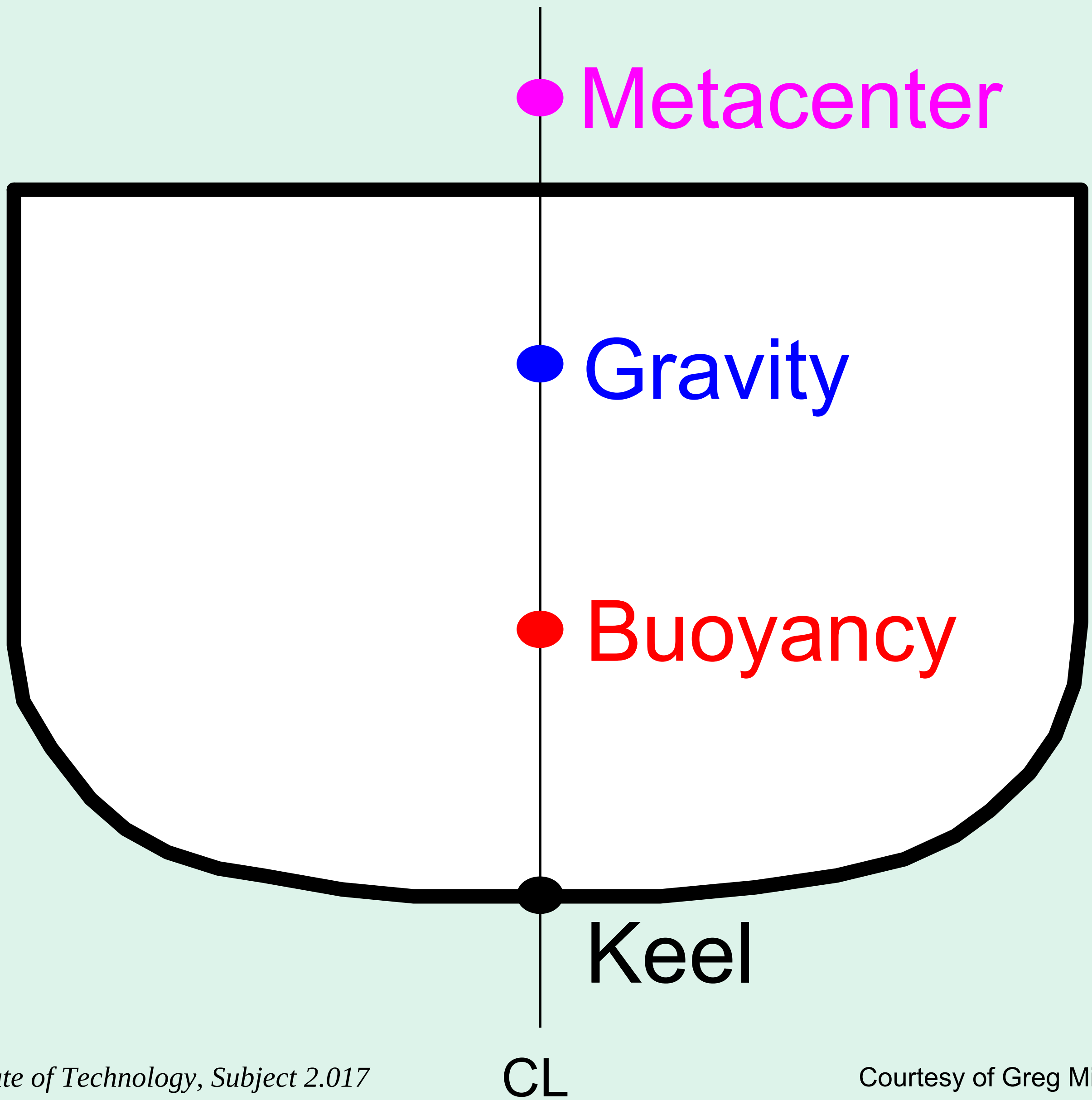
ζ = 0.5 gives about 15% overshoot

ζ = 1.0 gives the fastest response without overshoot

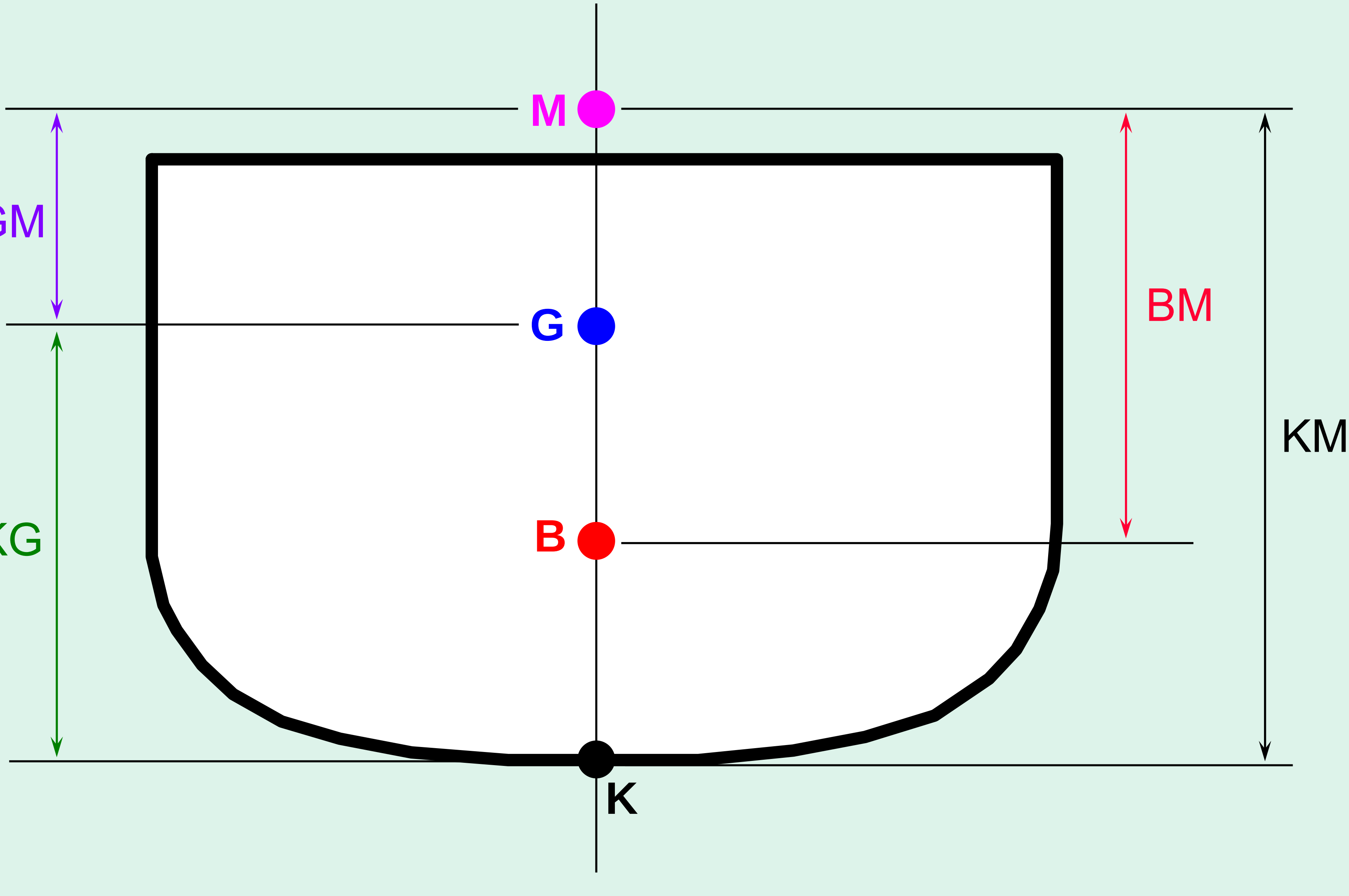
ζ > 1.0 is slower



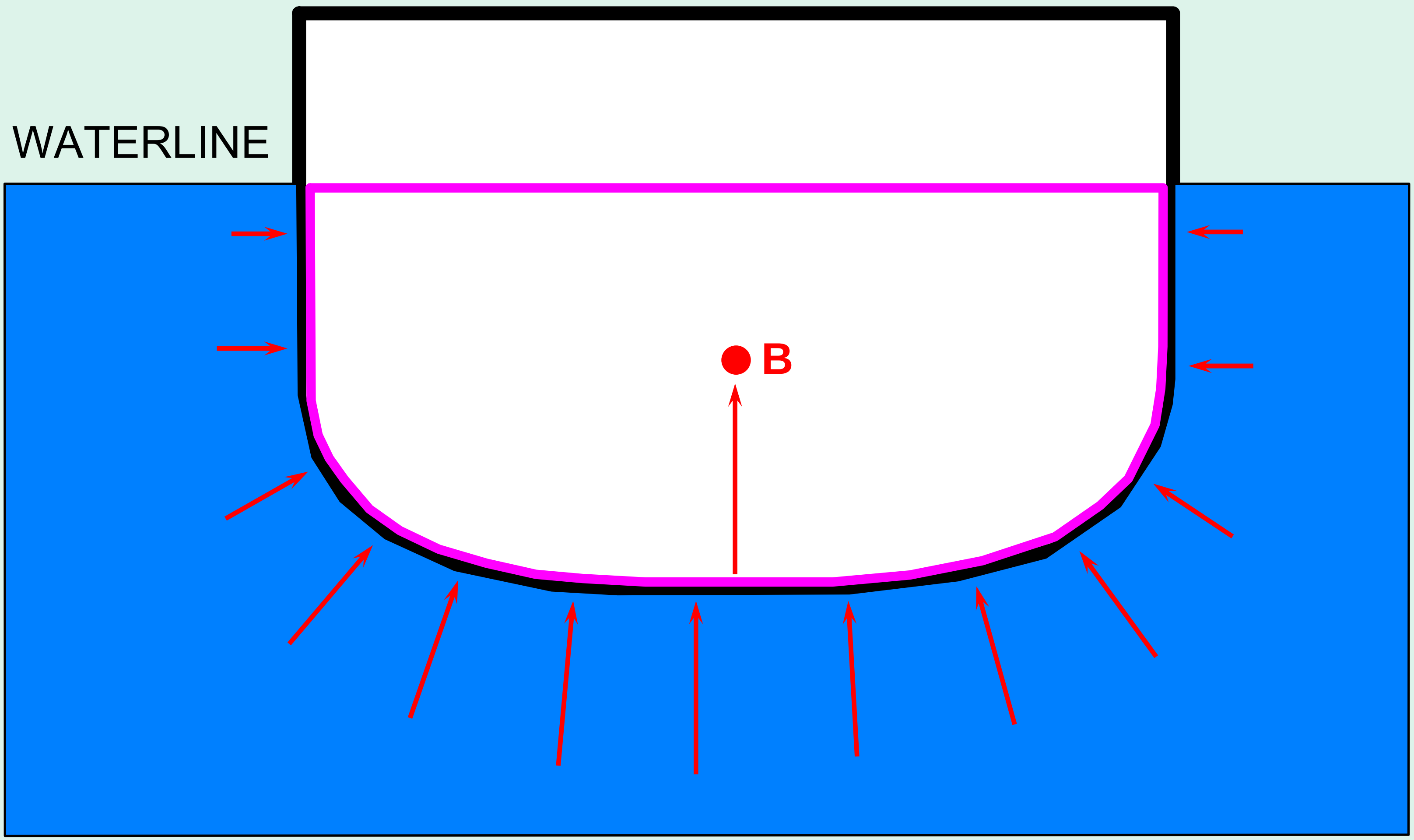
STABILITY REFERENCE POINTS



LINEAR MEASUREMENTS IN STABILITY



THE CENTER OF BUOYANCY



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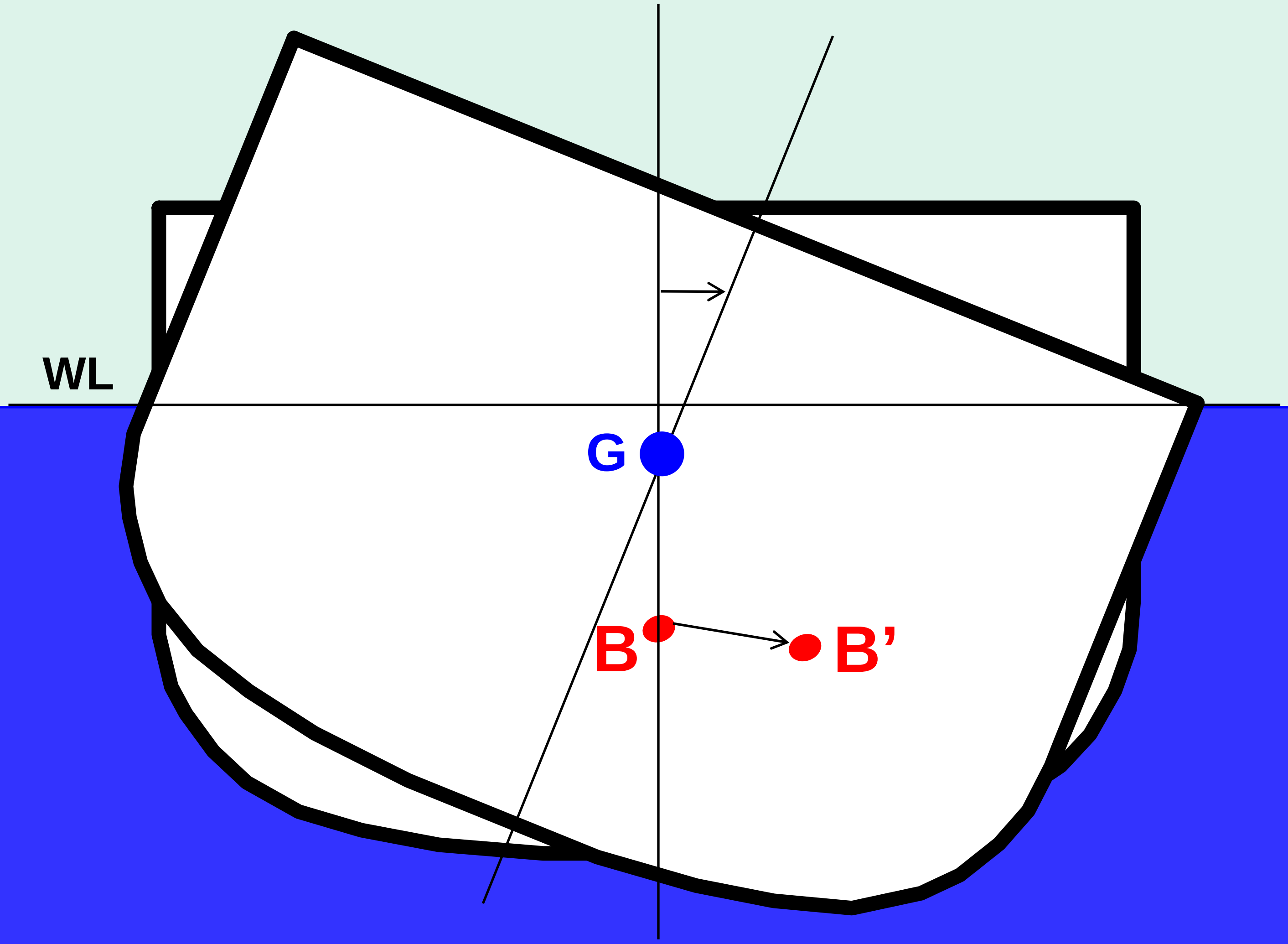
CENTER OF BUOYANCY



CENTER OF BUOYANCY

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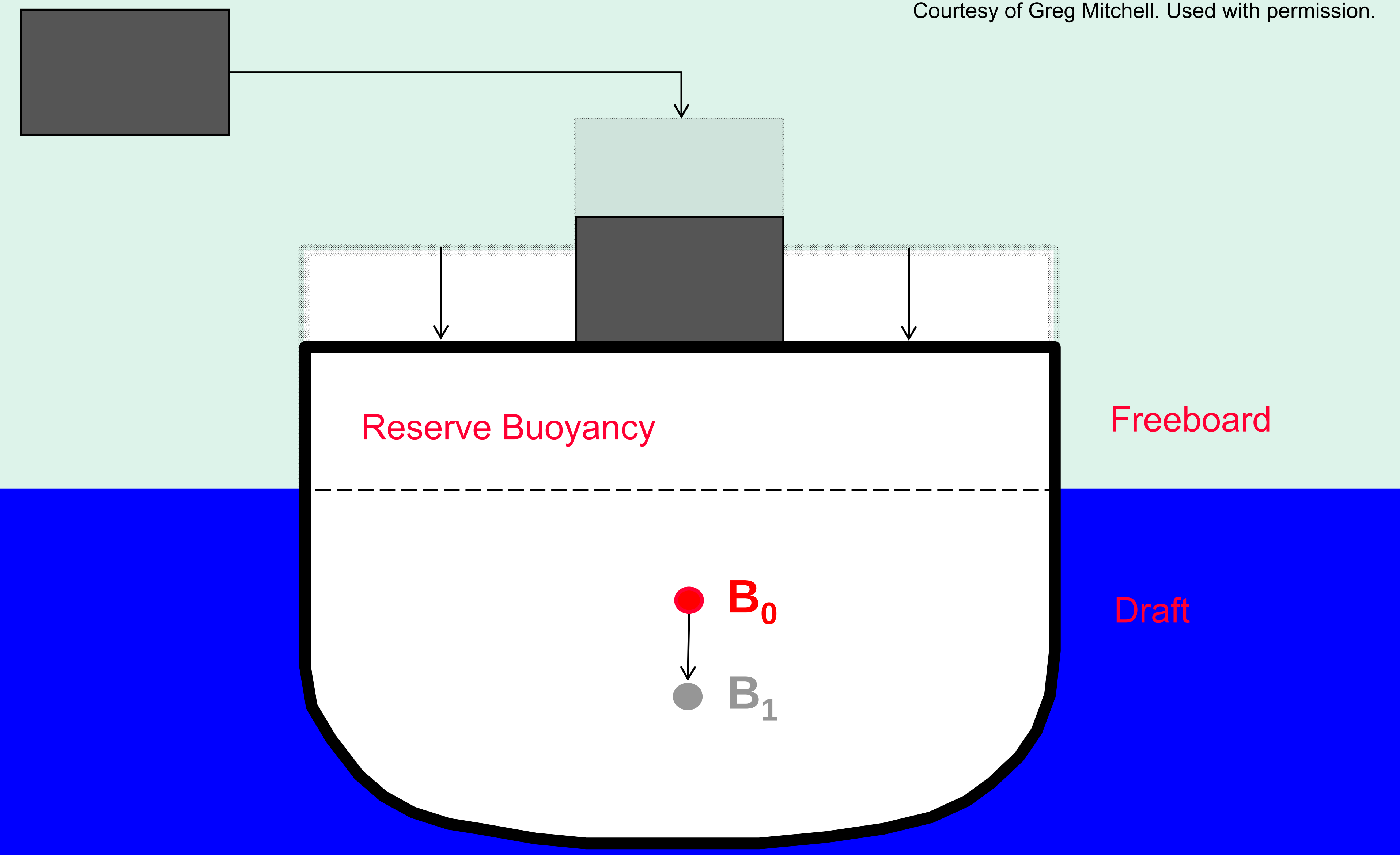
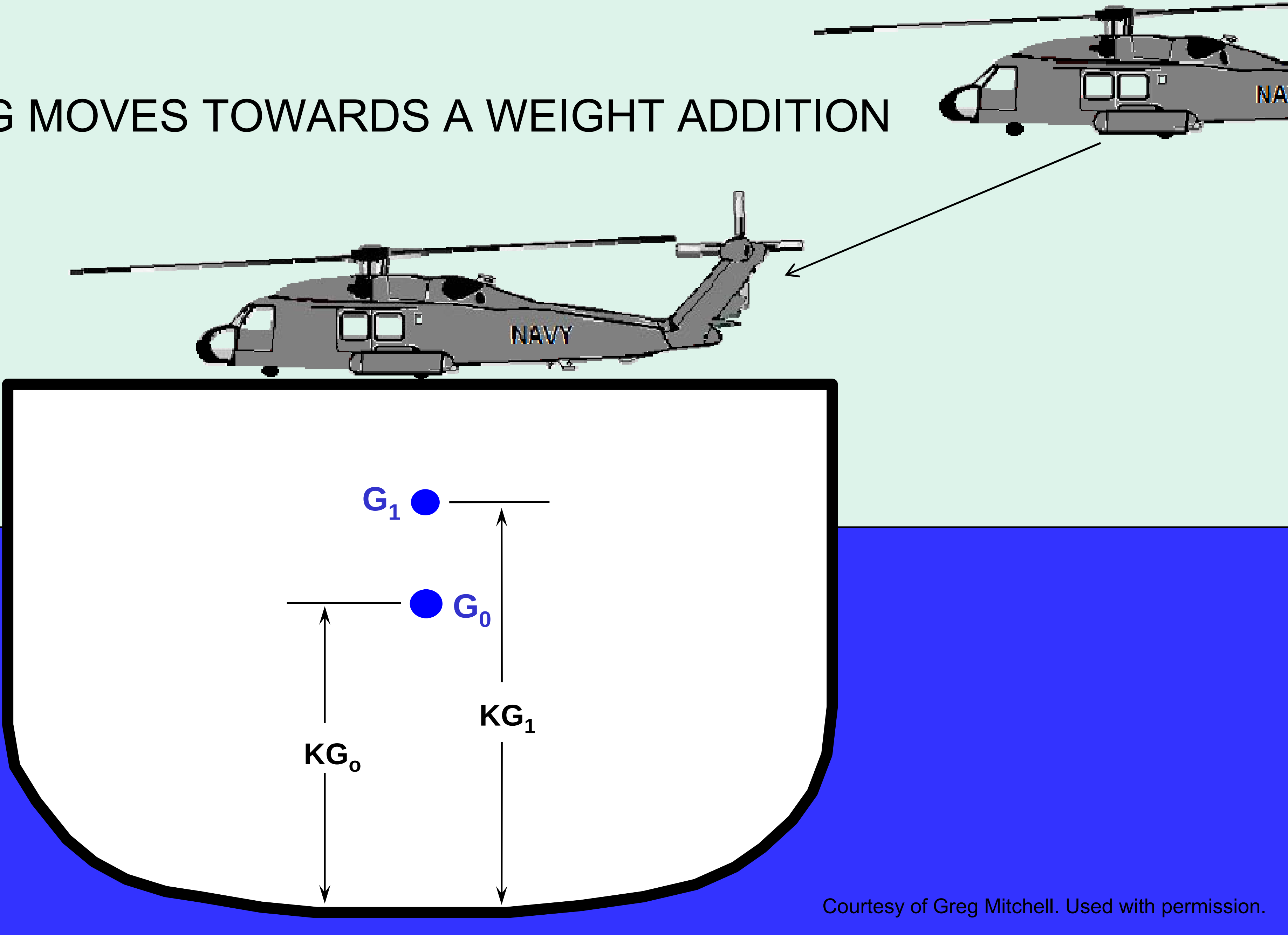


- The freeboard and reserve buoyancy will also change

MOVEMENTS IN THE CENTER OF GRAVITY

G MOVES TOWARDS A WEIGHT ADDITION



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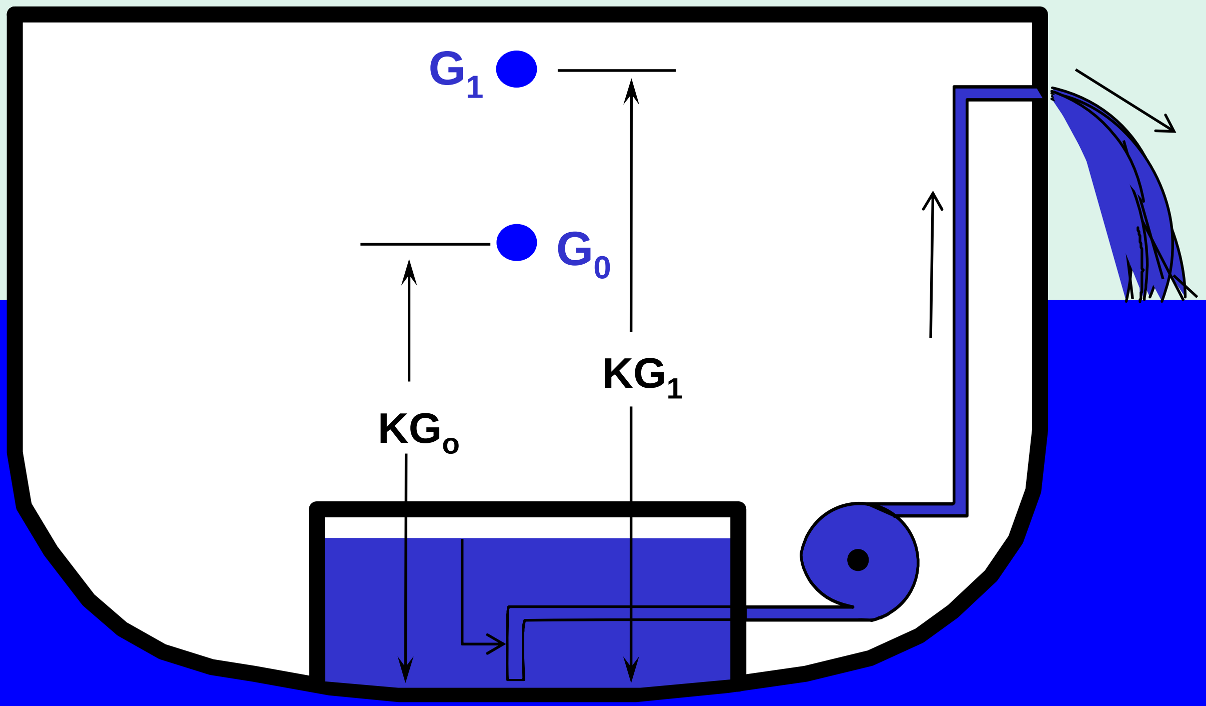
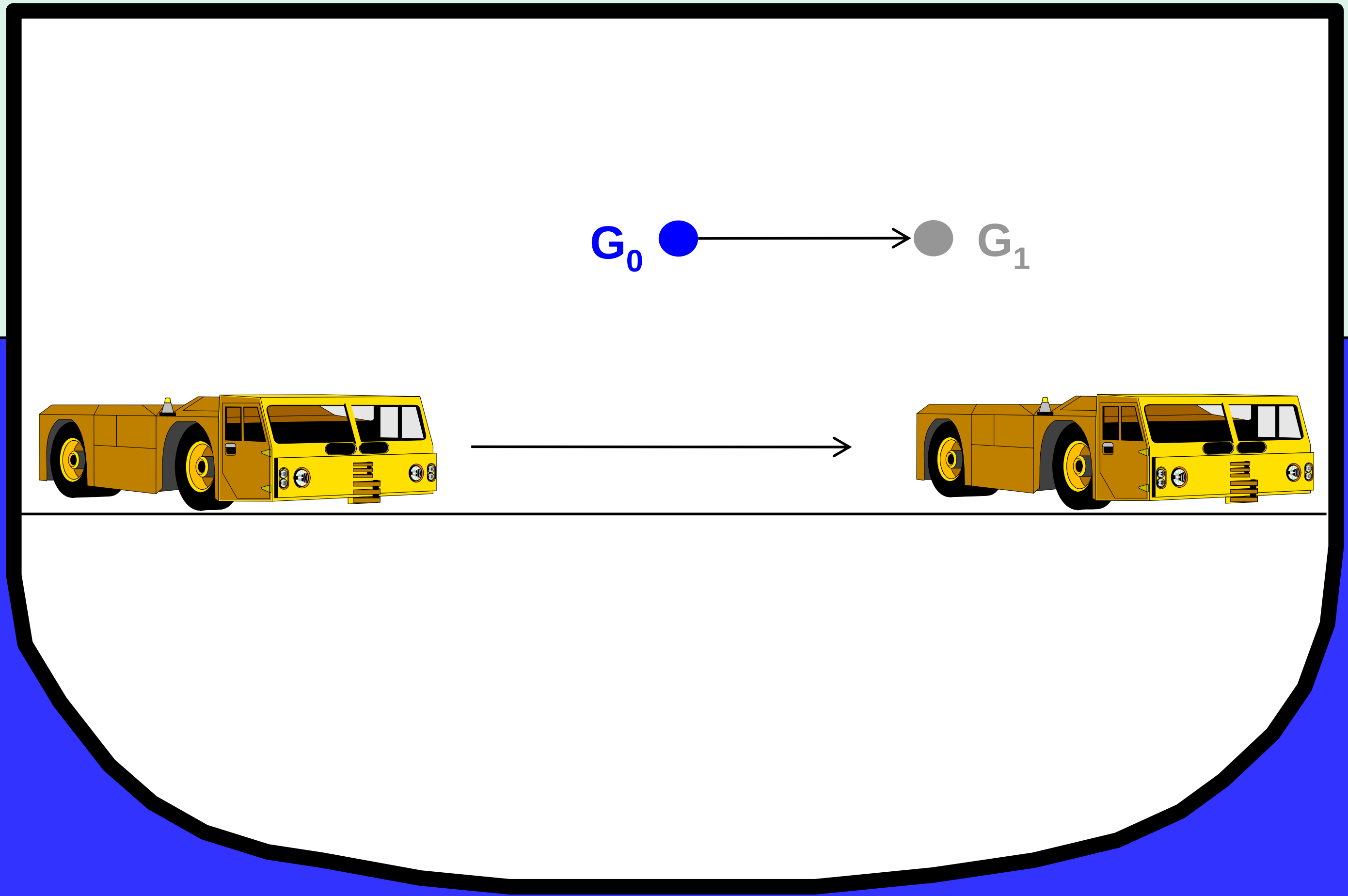
MOVEMENTS IN THE CENTER OF GRAVITY

G MOVES AWAY FROM A WEIGHT REMOVAL

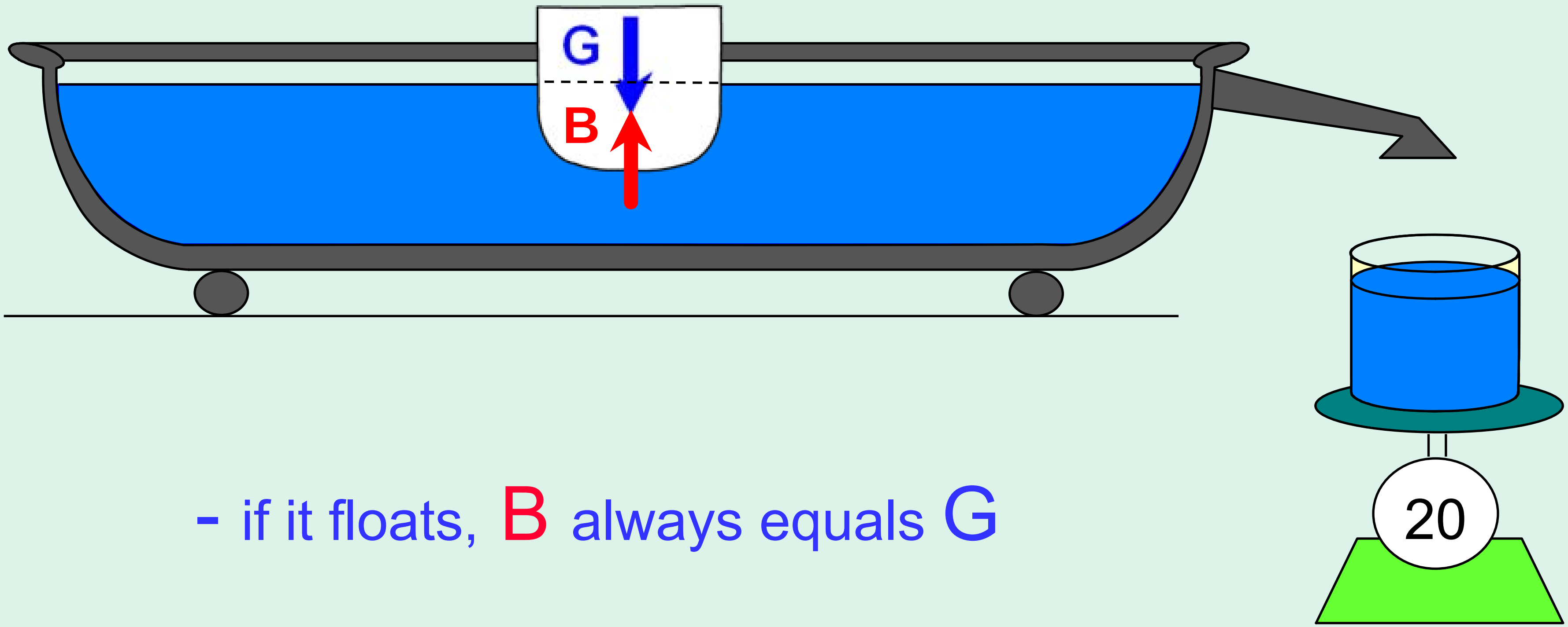


MOVEMENTS IN THE CENTER OF GRAVITY



G MOVES IN THE DIRECTION OF A WEIGHT SHIFT

DISPLACEMENT = SHIP'S WIEGHT



- if it floats, B always equals G

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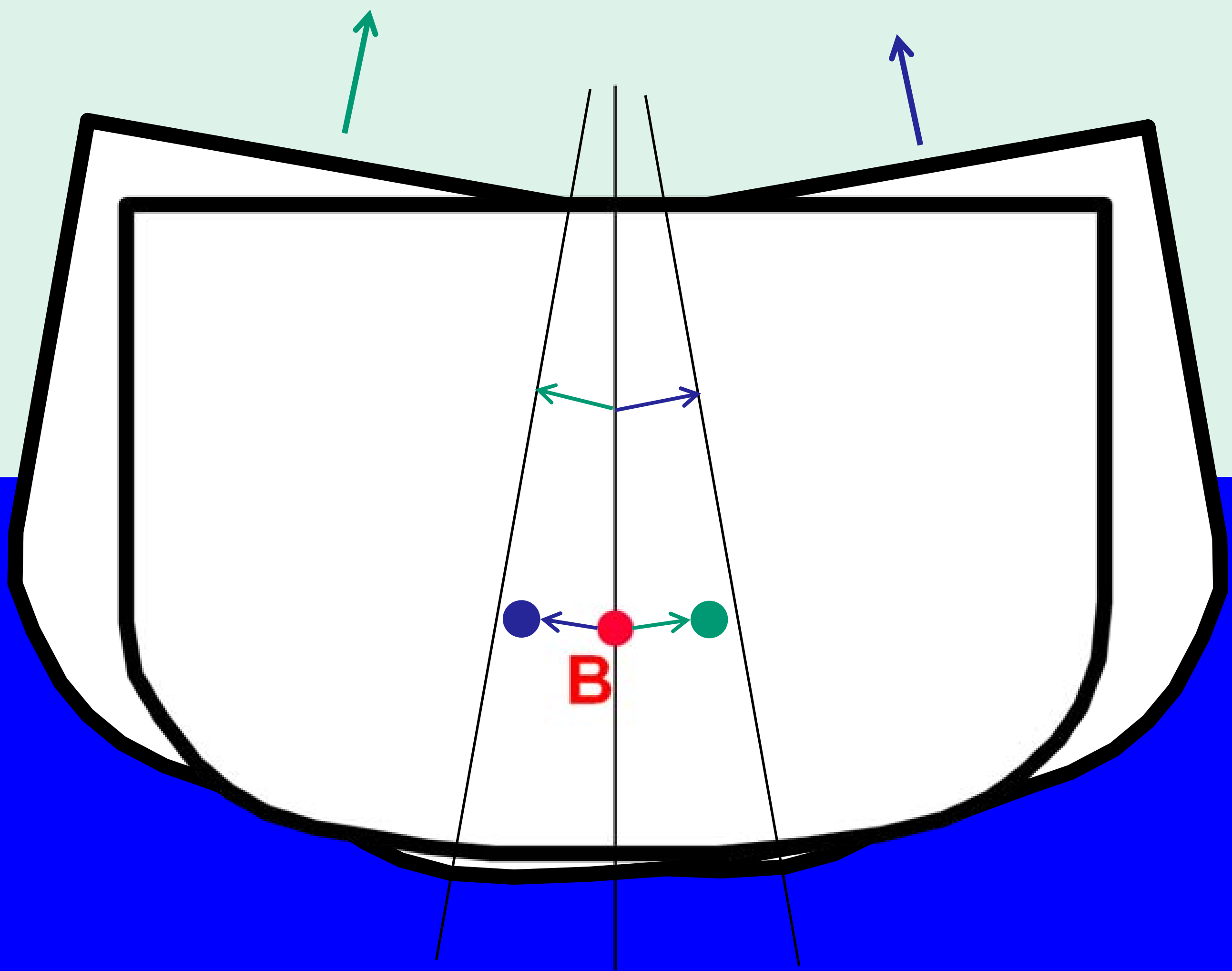
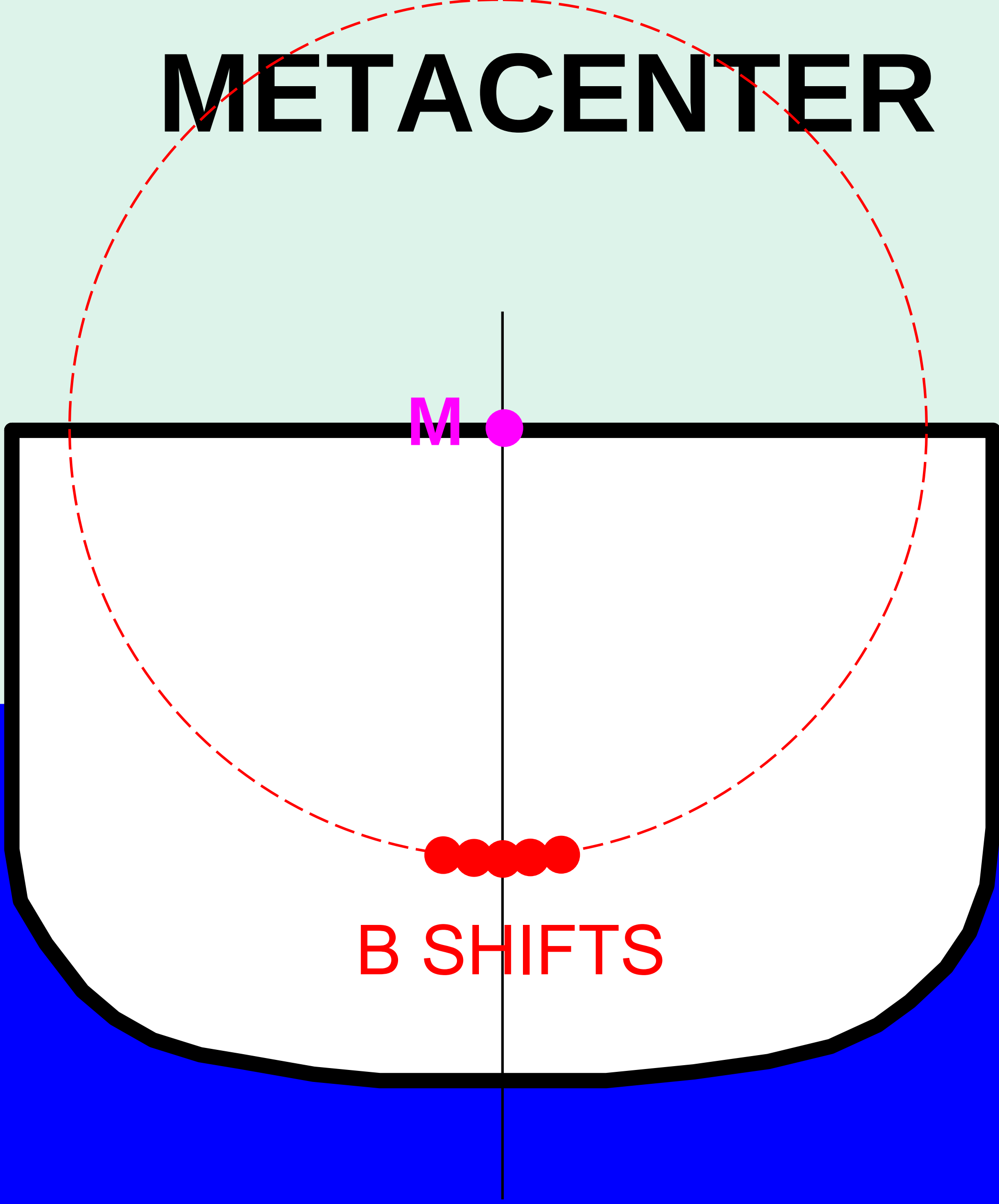
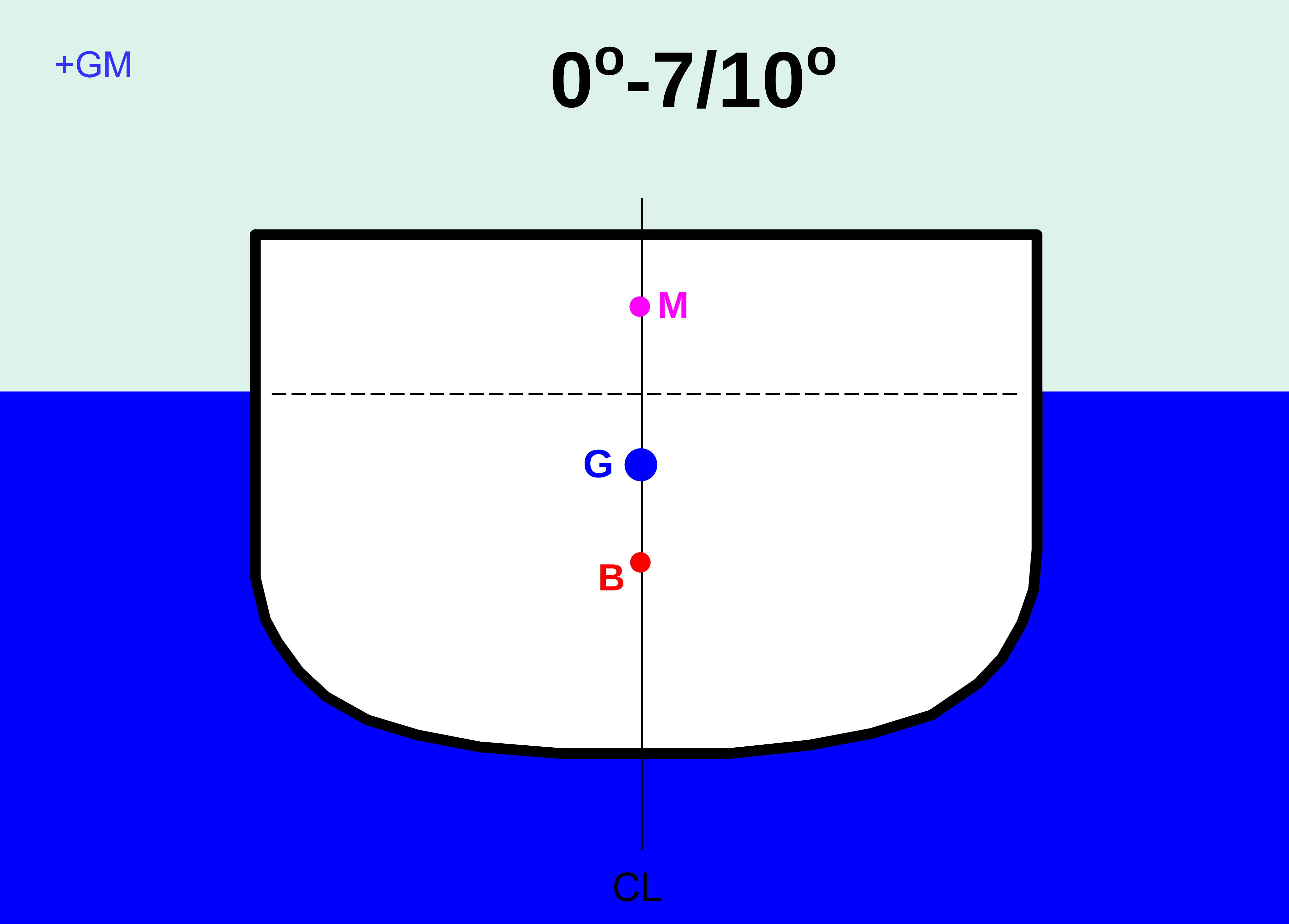
METACENTER

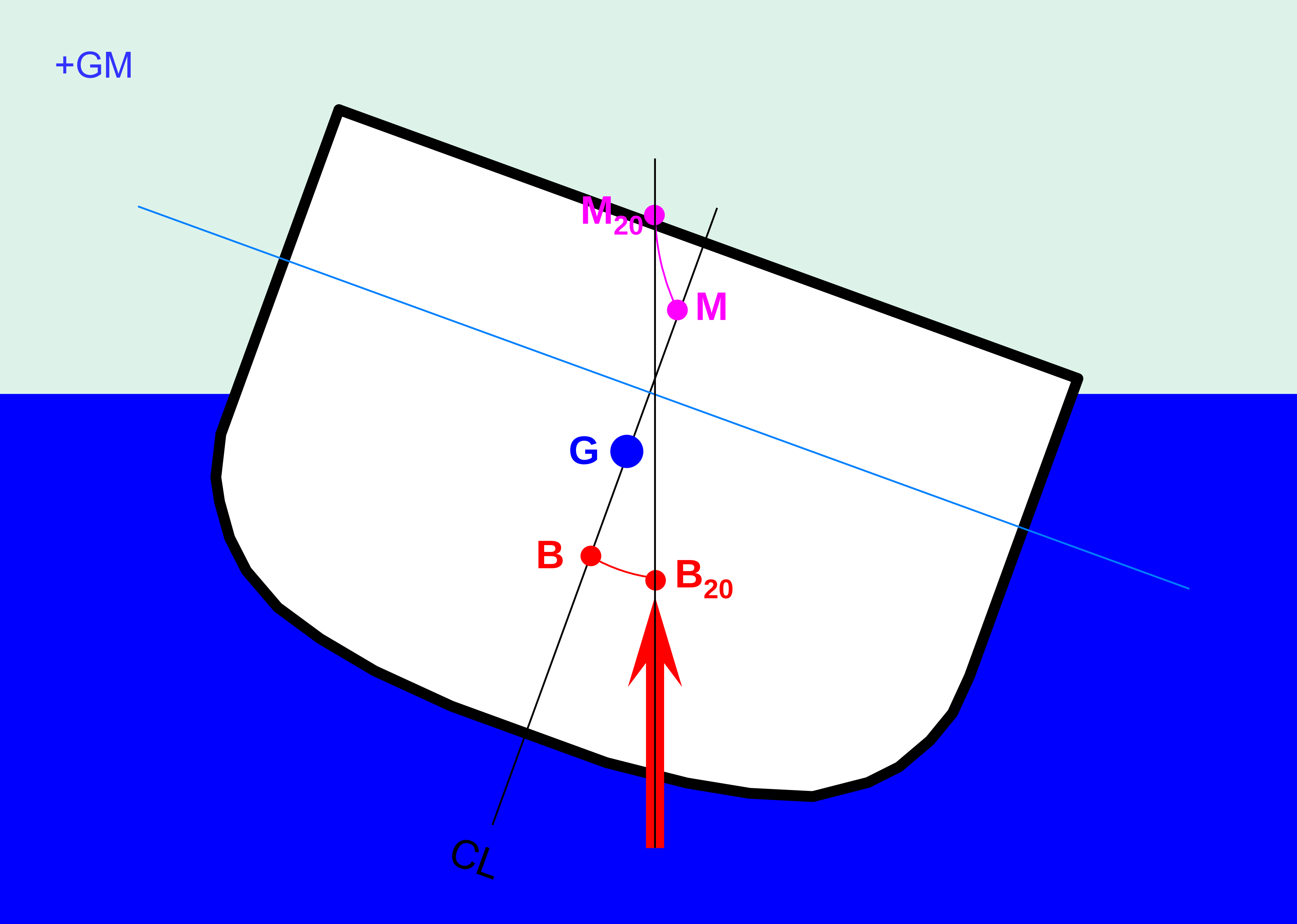


METACENTER



0°-7/10°





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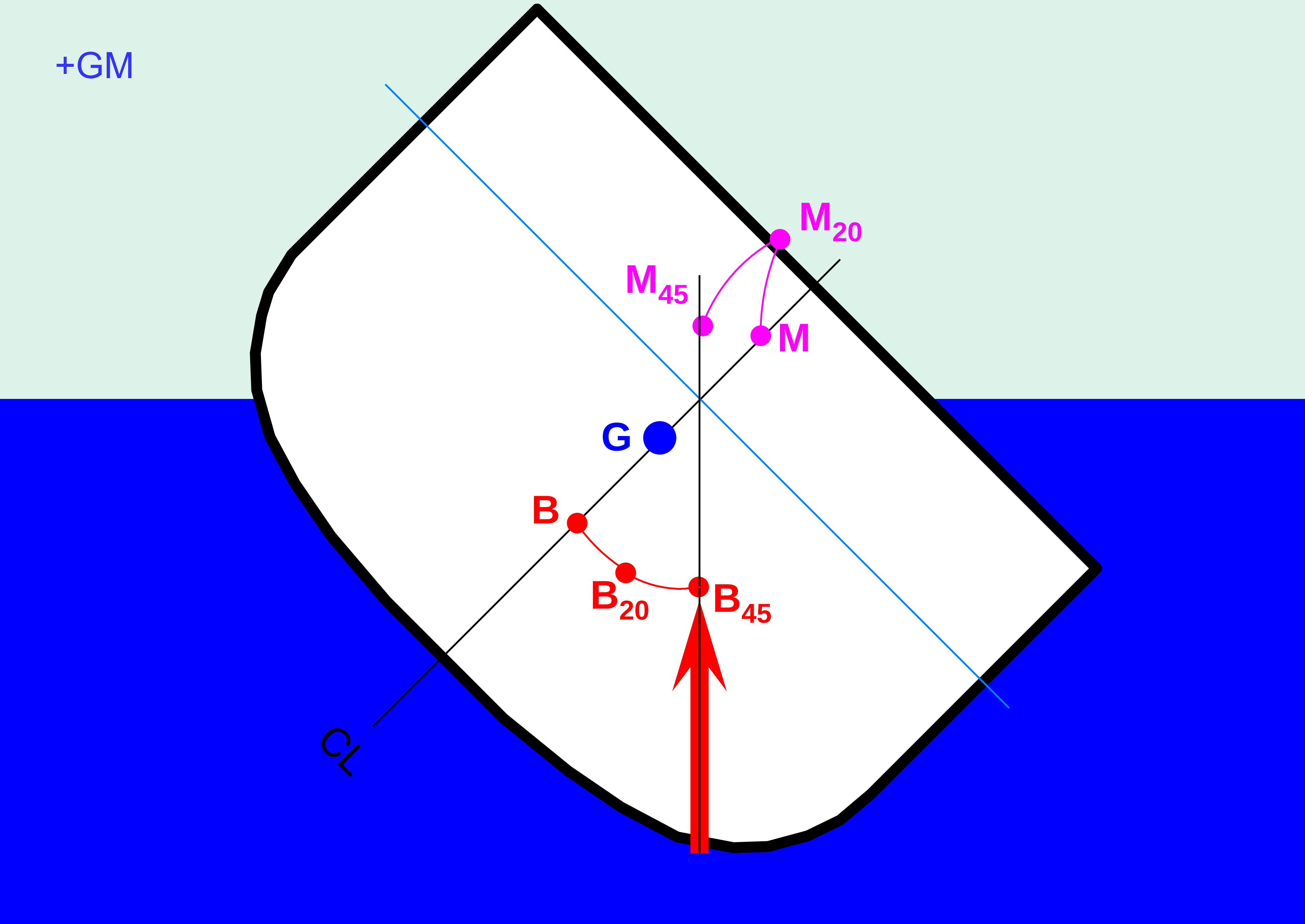
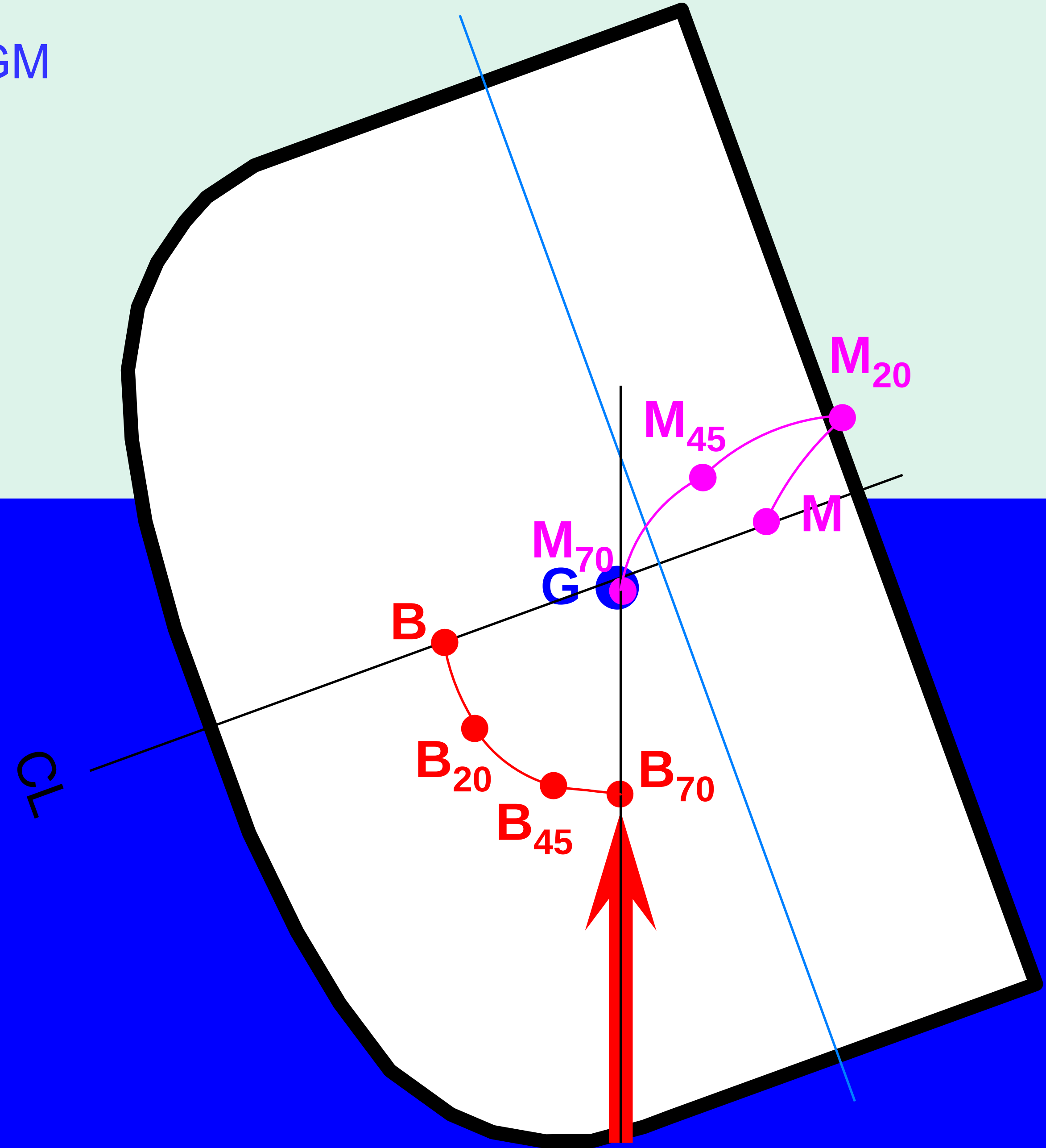

+GM
M20
M45
M
G
B
B20
B45
CL

neutral GM



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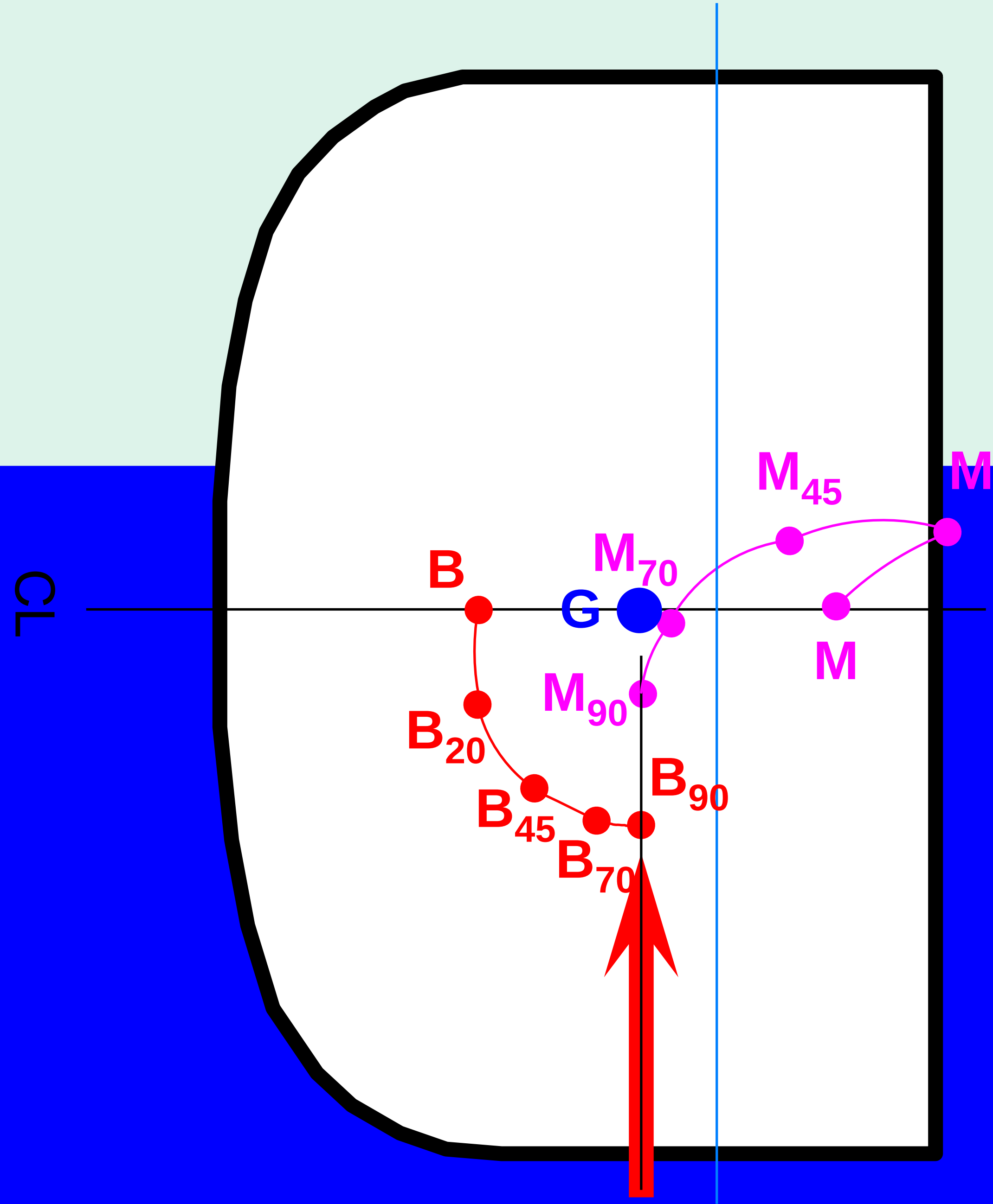
-GM

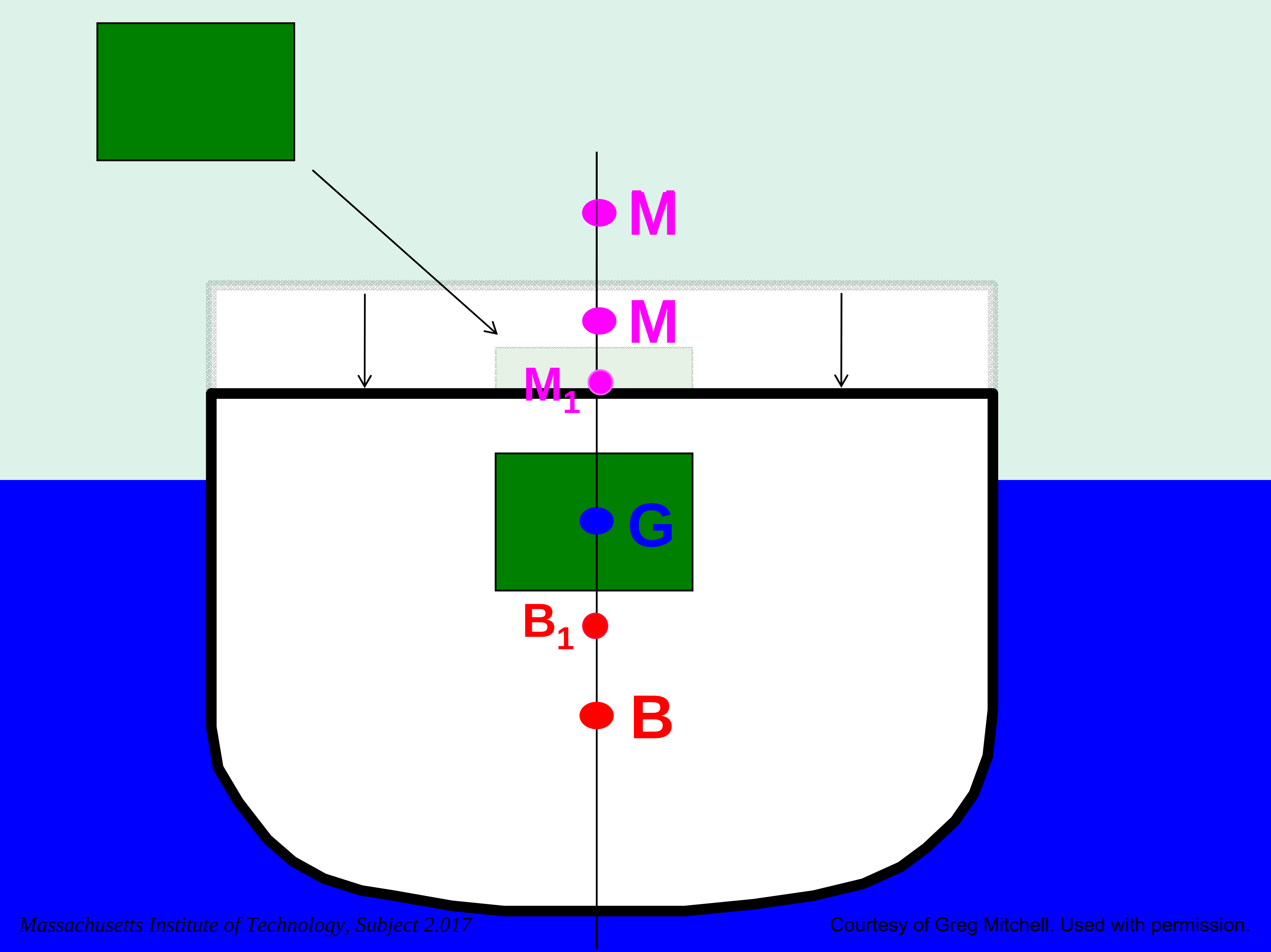


MOVEMENTS OF THE METACENTER

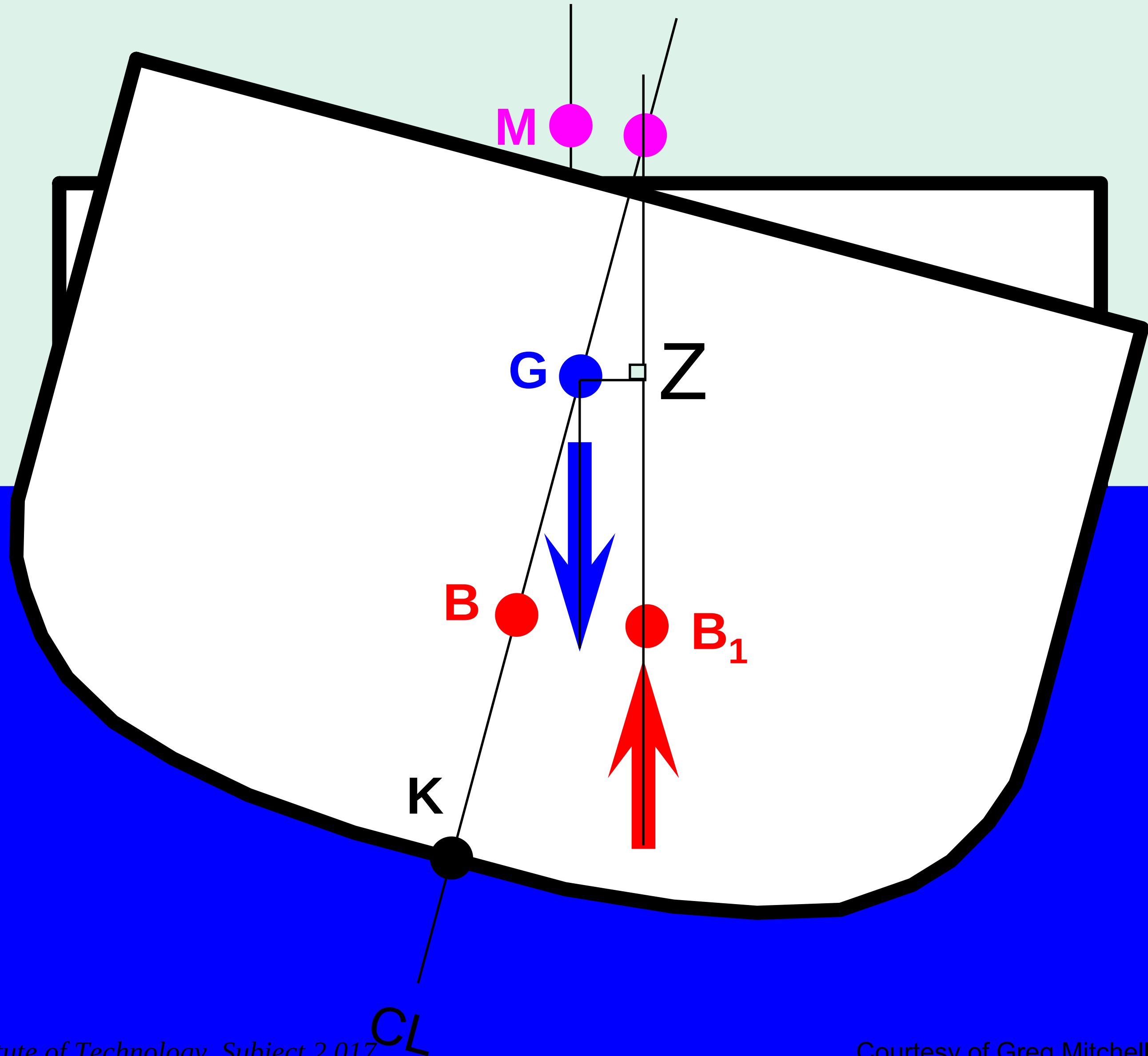
THE METACENTER WILL CHANGE POSITIONS IN THE VERTICAL PLANE WHEN THE SHIP'S DISPLACEMENT CHANGES

THE METACENTER MOVES IAW THESE TWO RULES:

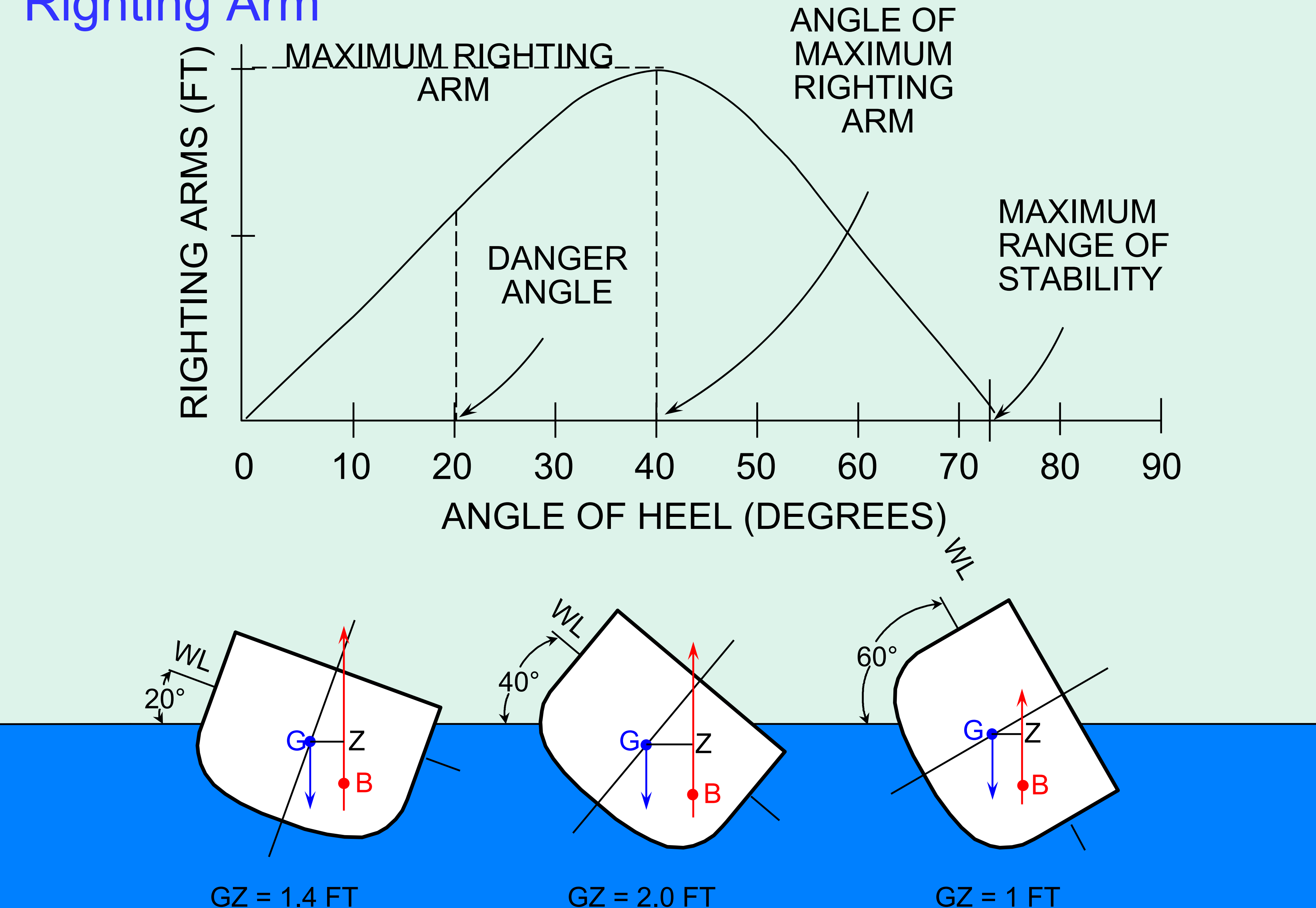
1. WHEN B MOVES UP, M MOVES DOWN.
2. WHEN B MOVES DOWN, M MOVES UP.


M
M
M1
G
B1
B

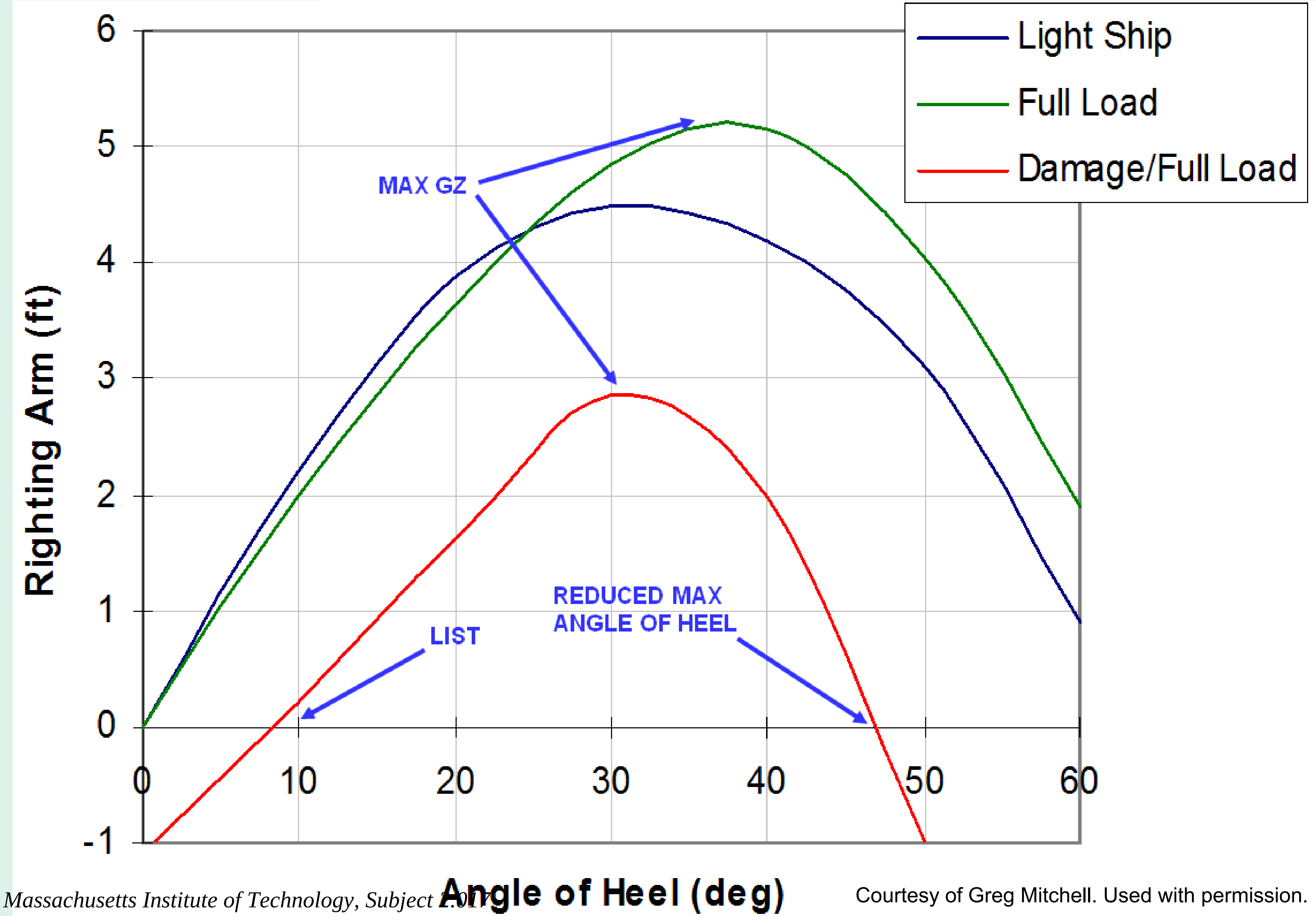
Righting Arm



Righting Arm



Righting Arm for Various Conditions



THINGS TO CONSIDER

- Effects of:
  - Weight addition/subtraction and movement
  - Ballasting and loading/unloading operations
  - Wind, Icing
  - Damage stability
    - result in an adverse movement of G or B
    - sea-keeping characteristics will change
    - compensating for flooding (ballast/completely flood a compartment)
    - maneuvering for seas/wind

References

- NSTM 079 v. I Buoyancy & Stability
- NWP 3-20.31 Ship Survivability
- Ship's Damage Control Book
- Principles of Naval Architecture v. I

2.017J Design of Electromechanical Robotic Systems
Fall 2009