SOLUTIONS
13.012 EXAM #2
FALL 2003

AVG = 76/95

PRINT IN LARGE LETTERS (Family Name) (First Name) (Middle Name) (Course) (Year)

(Subject Number) (Subject Name) (Date)

(Instructor's Name)

IF EXAMINATION FOR ADVANCED STANDING CHECK HERE ☐

IF A CONDITION EXAMINATION, FILL IN BELOW		IF A POSTPONED FINAL EXAMINATION, FILL IN BELOW	
(Year and term taken in Class)	(Instructor's Name)	(Year and term taken in Class)	(Instructor's Name)

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Do not write below. Reserved for Examiner.

(A) 1. $R_e = \rho U L/\mu$; $S_t = \frac{f a}{u}$; $C_D = \frac{D}{\frac{1}{2}\rho U^2 \ell^2}$; $a^* = \frac{a}{\ell}$; $d^* = \frac{d}{\ell}$

2. FROUDE = inertial/gravity = $U/\sqrt{gL}$; on ; just below

3. Tip speed to inflow speed $J = u/nd$

4. ⟶

X	0	0	0	0	0
0	X	0	0	0	X
0	0	X	0	X	0
0	0	0	X	0	0
0	0	X	0	X	0
0	X	0	0	0	X

5. $T = 111,111$ N

6. $M_{33} = \rho\pi a^2 L$; $M_{22} = 0$; $M_{66} = 0$; $M_{55} = \frac{\rho\pi a^2 L^3}{12}$

7. FROUDE #, weber #

8.

9.

10.

11.

12.

13.

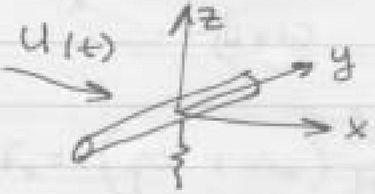
14.

15.

6-72 30M 016

PART B: SOLUTIONS

① $m^* = 85\ kg/m$



a) $f = 5\ Hz \qquad S_t = 0.2 = \frac{fd}{U}$

$$\frac{0.2}{5.0 \cdot 0.1} = \frac{1}{U} = 0.4\ s/m$$

$$U = 2.5\ m/s$$

b) $D = \frac{1}{2} \rho U^2 d L C_D$

$D/L = \frac{1}{2} \rho U^2 C_D d$

$D/L = 0.5(1000)(2.5)^2(1.2)(0.1)$

$\therefore D/L = 375\ N/m$

$C_D \rightarrow$ Re# dependent

$$Re = \frac{0.4\ m/s \cdot [illegible] \cdot 0.1m}{10^{-6}\ m^2/s} = 4 \times 10^4$$

$\therefore C_D = 1.2$

c) frequency of unsteady drag $= 2(\text{freq. vortex shed})$

$= 10\ hz$

Freq of lift $=$ freq. vortex shed

$= 5\ hz$

(2)

$$F_x = \int_{-(H+h)}^{0} dF_x$$

$$dF_x = (\rho V + m_a)\frac{\partial u}{\partial t}dz + \tfrac{1}{2}\rho u^2 C_D\, d(z)\, dz$$

Break into two parts based on diameters

$$F_x = \int_{-(H+h)}^{-h} dF_x + \int_{-h}^{0} dF_x$$

$$\frac{\lambda}{d} \gg 1 \;;\; \frac{h}{d} < \frac{2\times 0.6d}{d} = 1.2 > 1 \quad \left(1 < \frac{h}{d} < 10\right)$$

so complete form of Morrison should be used

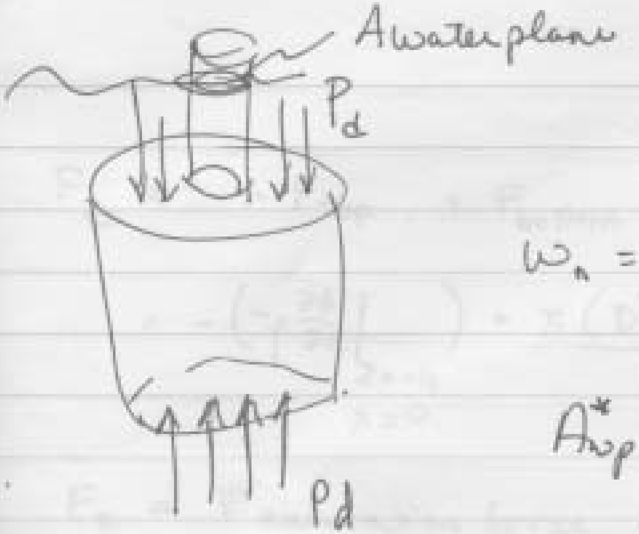
$$\frac{du}{dt} = \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial x}\right) = a\omega^2 e^{kz} \sin(kx - \omega t)$$

$$U^2 = U|U| = \left[a\omega e^{kz} \cos(kx-\omega t)\right]^2 + U_{current}$$

$$\frac{\partial \phi}{\partial x}\left|\frac{\partial \phi}{\partial x}\right| =$$

C_D either 0.3 for $Re \geq 3\times10^5$ or 1.2 $Re < 3\times10^5$

Force is probably less on legs that feel the current last. The "front" legs will disturb the flow, shedding vortices, & slow the local flow down so U↓ in the second term.



$$\omega_n = \sqrt{\frac{\rho g A_{wp}}{m + m_a}}$$

$A^*_{wp} = \frac{\pi d^2}{4}$ one pontoon; total area $A^T_{wp} = 4 \cdot \frac{\pi d^2}{4}$

$$m_a = 2 \cdot m_{a\,sphere} \text{ w/ } d = D = 2 \cdot \frac{1}{2} \cdot \frac{4}{3}\rho \pi r^3$$

$$m_a = \frac{4}{3}\rho\pi\frac{d^3}{8} = \frac{1}{6}\rho\pi d^3$$

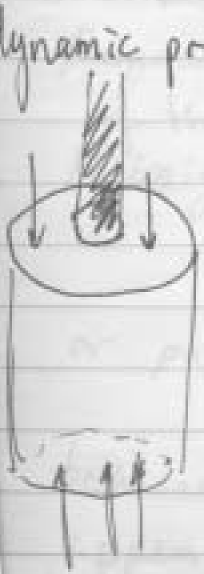
$$\Longrightarrow \omega_n = \frac{1}{2}\sqrt{\frac{\rho g \pi d^2}{m + \frac{1}{6}\rho\pi d^3}}$$ one pontoon

$$\omega_{n\,system} = \sqrt{\frac{\rho g 4(\pi d^2/4)}{4m + 4m_a}} = \frac{1}{2}\sqrt{\frac{\rho \pi g d^2}{m + m_a}}$$

(m = weight of displaced water $= \rho g \forall$)

F_z = (ΔPressure from top to bottom) Area

dynamic pressure $p_d\big|_z = -\rho \frac{\partial \phi}{\partial t}\Big|_z$ dependent on depth!



Top Area $= \frac{\pi(D^2 - d^2)}{4}$

Bottom $= \frac{\pi D^2}{4}$

$$F_z = -F_{top} + F_{bottom}$$

$$= -\left(-\rho \frac{\partial \phi}{\partial t}\Big|_{\substack{z=-h \\ x=0}}\right) \cdot \pi \frac{(D^2 - d^2)}{4} + \left(-\rho \frac{\partial \phi}{\partial t}\Big|_{\substack{z=-(h+H) \\ x=0}}\right) \frac{\pi D^2}{4}$$

F_z = $F_{excitation}$ force in heave.

d) $(m + m_a)\ddot{z} + b\dot{z} + cz = F_z(t)$

- $(m + m_a)$ ↑ mass + add mass ↓ 2 · ½ mass of sphere
- b ↑ damping
- c ↑ spring, $\rho g A_{wp}$
- $F_z(t)$ ↑ from part c

~~[illegible]~~ It will heave at the frequency it is excited at. (ie the frequency of the waves)

e) Minimize force → $F \sim P_d \cdot Area$
small area of bottom pontoon

or put pontoon deeper than $\lambda/2 \Rightarrow P_d \sim e^{kz}$

both are hard to design around. Could add appendages to increase damping....