

HW# 4 Solution Set

Problem 1: (10 points)

- a. (2 points) The **decibel (dB)** is a measure of the ratio between two quantities, and is used in a wide variety of measurements in acoustics, physics and electronics. While originally only used for power and intensity ratios, it has come to be used more generally in engineering. The decibel is widely used as a measure of the loudness of sound. It is a "dimensionless unit" like percent. Decibels are useful because they allow even very large or small ratios to be represented with a conveniently small number. This is achieved by using a logarithm.

$$I_{dB} = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

Intensity is I and pressure is denoted by P.

$$P_{dB} = 10 \log_{10} \left(\frac{P}{P_0} \right)$$

- b. (3 points)
- Jet engine – 130 dB @ 30 m
 - Car – motorcycle accelerating @ 5m = 110 dB
 - Whale – Blue Whale humming at 1m = 180 dB
 - Underwater Seismic Activity - seafloor volcanic eruption = 255 underwater dB @ 1m.
 - Rock concert – average is about 100 dB
 - Ships propeller – Supertanker = ~127 dB in air and 190 dB in water
 - Dolphin - 208 dB in water @ 6 m.
- c. (3 points)
- $10 \log_{10}(A/B) = 10 * (\log_{10}(A) - \log_{10}(B)) = \log_{10}(A^{10}) - \log_{10}(B^{10})$
 - $10 \log_{10}(A+B) = \log_{10}((A+B)^{10})$
 - $10 \log_{10}(A^B) = B * 10 * \log_{10}(A)$
- d. (2 points)
- $10 \log_{10}(1) \text{ dB} = 0 \text{ dB}$
 - $10 \log_{10}(10) \text{ dB} = 10 \text{ dB}$
 - $10 \log_{10}(0.5) \text{ dB} = -3.01 \text{ dB}$
 - $10 \log_{10}(4) \text{ dB} = 6.02 \text{ dB}$
 - $10 \log_{10}(16) \text{ dB} = 12.04 \text{ dB}$

Problem 2: (10 points)

- a. (3 points) A sound wave is a pressure wave traveling at the speed of sound in saltwater at ~1500 m/s and in air at ~343 m/s
- b. (4 points) using saltwater
- air = 343 m, saltwater = 1500 m
 - air = 1.715E-4 m, saltwater = 7.5E-4 m
 - air = 7.6E-4 m, saltwater = 3.3E-3 m
 - air = 34.3 Hz, saltwater = 150 Hz
 - air = 171.5 Hz, saltwater = 1500 Hz
 - air = 3.43 Hz, saltwater = 15 Hz



- c. (3 points) Higher frequencies attenuate more poorly than lower frequencies in all three mediums, so the lower frequency (i) will travel the furthest or attenuate the least. Sound waves travel fastest in steel, then water, then air.

Problem 3: (2 points)

- a. (2 points) Passive sonar only listens to sounds (receiver), whereas Active sonar sends and receives signals to determine distances, shapes etc.
- b. (2 points) $P_{\text{ref}} = 1 \mu\text{Pa}$
- c. (2 points) Directivity determines how poorly the signal is attenuated in a certain direction. If the signal is not directed at all, then the signal is attenuated equally in all directions, however, if the source is focused in a certain direction then it will attenuate much less in that direction and will be more intense in the direction it is intended. Directivity is used to determine if a receiver is within the focus direction or inside a null node or side band signal etc, and can help determine the signal strength.
- d. (2 points) $DI_{\text{line array}} = 10 \log(2L/\lambda)$ and $DI_{\text{rectangular array}} = 10 \log(4\pi L_x L_y / \lambda^2)$ so the line array only considers the length, where the rectangular array must consider the length and width, as well as dividing by wavelength squared.
 $DI_{\text{line}}/DI_{\text{rectangle}} = \lambda/2L$.
- e. (2 points) The line array on the left has a very large intensity at its center in the horizontal direction; the line array on the right has a large intensity along its vertical centerline. John Leonard pointed out that the longer an array is the narrower its beam is concentrated near its center.

Problem 4: (10 points)

- a. (5 points) Sound waves tend to bend towards regions with low velocities. In the upper mixed layer the sound tends to move towards the surface. In the upper layer the sound waves cannot travel more than a few kilometers. If it is reflected from the surface it tends to bend back to the surface as it moves through the region. In the thermocline the sound waves tend to bend toward the region of slowest velocity. When a sound is transmitted it is forced out in all directions (unless specifically directed towards on direction) as it travels near a higher velocity it is bent back to the lower speed region. In the sound channel the sound waves can travel thousands of kilometers.
- b. (5 points) The sound waves will tend to travel in large sinusoidal waves, not in a horizontal line, because of the nature of the bending of the waves as they move through different sound speed regions. In general they always travel with some sinusoidal propagation.

Problem 5: (10 points)

- a. (4 points) $p(r, t) = \frac{A}{r} \cos(kr - \omega t) = \text{Re} \left\{ \frac{A}{r} e^{-i(kr - \omega t)} \right\}$
- $$I(r, \theta, \phi) = \frac{1}{\rho c} \left[\frac{1}{T} \int_0^T p^2(r, \theta, \phi, t) dt \right]$$



$$\text{Definition of RMS: } f_{rms} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} [f(t)]^2 dt}$$

$$p_{rms_{r_1=0}} = \sqrt{\frac{1}{T} \int_0^T p^2(r, \theta, \phi, t) dt}$$

$$I(r, \theta, \phi) = \frac{1}{\rho c} p_{rms}^2$$

b. (2 points) $P(r) = \int I(r, \theta, \phi) d\Omega$

$$P(r) = \int_{\phi=-\pi}^{\pi} \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\rho c} p_{rms}^2 r^2 d\phi \cos(\theta) d\theta$$

$$P(r) = \frac{4\pi}{\rho c} r^2 p_{rms}^2$$

c. (2 points) $P(r)_{r=1m} = \frac{4\pi}{\rho c} p_{rms}^2$ For the source level.

$$SL = 10 \log(P) = 10 \log\left(\frac{4\pi}{\rho c}\right) + 10 \log(P_{ref}^2) + 10 \log\left(\frac{P^2}{P_{ref}^2}\right) = -50.8 - 120 + SL$$

$$\therefore SL = 171 + 10 \log(P)$$

- d. (2 points) The SL at 1m has an easier equation because the r-value is 1. It is also a less noisy signal so the dominant source at that distance, so its power output is higher than any surrounding signal.