PROFESSOR: Well let's see if we can't get started. Everyone I trust can hear me adequately. Welcome back. It's Tuesday.

For those of you who are not in my recitation section, I'm Dave Gossard, and I'll be your lecturer for the day. Professor Vandiver is out of town. It looks like some of you may be as well. We probably could have held this at the gate at Logan Airport and done a little better.

But be that as it may, glad you came. This should be fun. Today we have a new topic and a demonstration, a real physical system. So unless there are any outstanding questions? Anybody have any questions or complaints to address to Vicente?

No. All right, hearing none let's go ahead and get started then. Today the topic is multiple degree of freedom systems. Now to date, with a couple of exceptions, all of the systems that you've dealt with had a single degree of freedom, either a linear displacement x or an angular displacement theta.

You know the concept of equations of motion, or I should say the equation of motion and the notion of undamped natural frequency. Well, today we're going to generalize, if you will, to systems that have not one but multiple degrees of freedom and see how those notions generalize. In particular, as you might expect, the system that has multiple degrees of freedom has multiple natural frequencies, also known as eigenvalues as we will explain here shortly.

Multiple degrees of freedom systems have a new property, a new characteristic you haven't seen before. And that's what this lecture is all about, is to illustrate that to you and demonstrate it. It's the notion of natural modes, also called eigenvectors.

And then the general response to initial conditions.

So that is the plan for the day. And we'll start with this. This is kind of a classic textbook case, two springs, two masses. A straightforward extrapolation of what you've done before.

You've got a spring K1, mass M1, spring K2, mass M2. And the displacements are indicated as shown there, X1 and X2. I want to hasten to point out that the displacements we speak of here are defined with respect to the static equilibrium position.

This is a notion that Professor Vandiver went over at least once. And for those of you who've forgotten it or weren't there that day, I have for you a reference, essentially reprised that notion over there. So in the meantime, let me press on.

If you have any questions, we can go back and cover that. But assuming you agree, let me simply say you've got two springs, two masses. The typical way we've taught you to do it is if you're going to generate the equations of motion by the direct method, you generate two free body diagrams, the sum forces in the x direction for each of the masses, get f equals MA and you'd get these.

Conversely, you could also do it by Legrange. You could generate the expression for the kinetic energy, for the potential energy, for the Legrangian, do this Lagrange equation business, and you'd get the same thing. But either way you do it, what comes out the other side looks like this. And it's not a bad exercise for you to offline convince yourself that this is right. Not right now, but in the comfort and leisure of another time.

So there you have it. That's what the two equations of motion would look like. Again, either by the direct method or by Legrange, you end up in the same place, so to speak. And now for today, we haven't asked you to do this much, but let me simply say the weapon of choice for multiple degrees of freedom system, because there's a certain repetitive quality to it, matrix notation is preferred. In these equations over here, written in matrix form would look like this.

And that looks like this. There's two matrices, and let me hasten to point out that this is exactly this and nothing more. There's no magic, no additional derivation. This is simply a restructuring, and reorganization of these equations.

And this may seem foreign to those of you who have not had any or very much linear algebra. Do not be dismayed. It is not a difficult thing to learn. As you probably know, a matrix multiplies by a vector-- or multiplying a vector by a matrix-- is done with two hands.

The first item for example is M1 X1 dot plus 0. That gives you this term right here. Over here you get X1 times K1 plus K2. That's this one.

And here you have minus K2 X2, that's that term. So matrix notation, this becomes that. No problem.

It's like a model train set. It's great. Everyone should have one of these.

All right, so what I would like to do for our example here is because we're going to be doing some algebra, for the express purpose of simplifying the algebra let me consider a special case where the masses are identical. And we can simply call them M. And similarly, the springs are identical, and we'll simply call them K.

At that point, these equations become simplified. That's just M, that's just M. So this is the problem we're going to-- we're going to tackle this problem first, because as I say, it simplifies the algebra. Now here is-- this is not an assumption.

This is a-- I would call this more a mechanism to get this job done here. Harmonic motion is one where we assume that the masses oscillate at the same frequency. So what this looks like is this. What we're basically saying is that X1 is actually equal-- X1 has amplitude A-- whoa. I'm being attacked here by the second board.

Basically, the situation here is that we're assuming that both of these masses move-- I wouldn't call it exactly together. They're not in complete synchrony, as you'll see here in a moment. But what they are is they're going through a sinusoidal motion. And it is an oscillation at the same frequency. Both of them are oscillating at the same frequency.

However, they differ in their magnitudes. They are not the same magnitude. But that assumption right there allows us to say this. If we differentiate those twice, we get the following. That comes back out.

Douglas, where's that minus sign come? Can you-- first of all, can everybody read that? Can you guys read that in the back row here? For example, this says that X1 double dot, if x1 is A1 cosine, then X1 double dot is A1 cosine preceded by a minus omega squared. Where does that come from?

AUDIENCE: The minus sign came from when you differentiated the cosine in the first one.

PROFESSOR: Exactly. And the cosine returns. And that's what you get. Well, here is, shall we say, the heart of the matter.

When you substitute this into this-- let me call this-- I'll try not to get too obsessive over this. But these are our equations of motion. So when we-- you get this, equals 0. Excuse me.

Many people, I think, simply put a big 0 there, but I'll do it properly. It's two zeros, if you will. And forgive me for writing this out, but I would like you to be able to do this by yourself, to recreate this after the fact.

This becomes-- dividing and collecting terms. OK, anybody unclear about how this is obtained? Yes, ma'am. Emma.

AUDIENCE: I have a question about the previous one.

PROFESSOR: Yeah?

AUDIENCE: If the second term on the left hand side, should it also be multiplied by A1 A2?

PROFESSOR: Yes it should. Thank you very much. Oh, hang on. Yes, thank you, that's exactly right.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Hang on a second. We're fighting the boards here. Let's see, one thing at a time.

We've got A1, A2 cosine [INAUDIBLE] minus phi equals. Now Emma, does that take care of you? Yeah? And you said, Vicente?

AUDIENCE: Diagonal terms-- shouldn't there be a minus?

PROFESSOR: I'm sorry, that's absolutely correct. Wonderful. So there we have it. Any other questions? I hope I got it right. Yes sir.

AUDIENCE: Are they plus K?

PROFESSOR: I'm sorry?

AUDIENCE: Are they plus K?

PROFESSOR: Plus K? No. They're minus. Yeah, why is that? Everybody see that?

This is a straight-- there's less here than meets the eye. There's a straight segregation collecting of terms. The minus got added to the elements of the mass matrix, but not to the K metrics. The K metrics goes shows through as is.

Now, the question on the floor is what we do with this? Can everybody appreciate that-- get out of the spring mass business and look at this from a math point of view? Does everyone appreciate that this is a set of linear equations? There's the old AX equal B kind of thing.

And if you recall, to solve, what we're basically going to do is solve for A1 and A2. That's the game we're playing here. And if you recall from your math course, the determinant of this has got to equal 0. So let me simply repeat it here. The determinant of has got to equal 0.

And you recall, the determinant is for at least the two by two you can do it by hand more or less. It's the cross products with appropriate sign. And I'm sparing you some algebra here, but trust me when you do this, this is what you get.

M squared omega 4 minus 3KM omega squared, plus K2. That's it right here. This little guy-- oh, all right. I can't do that anymore. I know, it's this one.

That is called the characteristic equation. So here's the first answer.

AUDIENCE: [INAUDIBLE] should that be K squared?

PROFESSOR: I'm sorry. That's a typo. That's simply K.

AUDIENCE: [INAUDIBLE] K squared?

PROFESSOR: Or it's K squared, rather. Sorry. Thank you.

AUDIENCE: I think in the above line, the determinant-- the upper left-- should had a 2K.

PROFESSOR: Oh, this is 2K, absolutely. All right, 2K. Good enough? All right, thank you.

So let's send this to the top. So the roots of the characteristic are the natural frequencies. Let's do this this way. Did I do that right? Yeah, plus or minus.

So the situation is that when you apply the quadratic formula to that characteristic equation to find the values of omega for which that equation is satisfied, those omegas that come out are the natural frequencies. They are the quantities we seek. And what that yields, as you can see from the plus or minus here, there are two of them.

I'll write the whole thing out here. And these are numerically. OK, everybody see that?

So here are our two natural frequencies. Here's the first one-- excuse me, that's not right either. That's the square. OK So these are our natural frequencies, once again for this special case where the masses are equal and the springs are equal.

Anybody recognize that number, 0.618, for all you fuss budgets? Ring any bells? Any number freaks here? No?

I heard it. That's it. Exactly, nice job. The golden mean, the golden ratio. Also, let me simply say if there are-- as far as the number of things-- if there are n degrees of freedom. There are n natural frequencies. What else? So that's that.

So now it's time to get to this notion of the natural modes. Let me say, we've got to go all the way back to this set over here. If you take the first row of this matrix equation-- that's the first of the equations of motion-- and you make that assumption of the harmonic motion in there.

Does everybody see that? What we've done is we've taken basically the first row of that expression right up there and formed the amplitude ratio A1 over A2. What we're doing is we've found the omegas.

You remember, just review the bidding. Our original assumption was harmonic motion, that is to say all the displacements are moving in synchrony as it were. The same sinusoidal frequency, we've just found what frequencies those are. There are two of them, and they're right there.

Now we're after these guys. Now we're after the relative magnitudes or the relative amplitudes of A1 and A2. And we from one of the equations isolated one of those. And let me just say, if you plug these back in, plug in the first one, you'll get oddly enough 1.618,

These amplitude ratios. Are the so-called natural modes. And I think you can appreciate that this is the first one, and this is the second one.

Any questions so far? Wonderful. Hearing none. Yes ma'am, Sara?

AUDIENCE: [INAUDIBLE]

PROFESSOR: You see this amplitude ratio. You saw how we got that. You see that the right hand side has got system parameter, Ks and Ms, and stuff like that. But this is the ringer, omega.

This amplitude ratio is expressed in part in terms of omega. So what omega-- there's no ambiguity as to Ks and Ms, but what omega? Well the answer is, when we plug in this one, you get this answer. When you plug-in this one, you get this answer.

So while we're at it-- Sara, want to hazard a guess? How many natural mode do you think we've got? Yeah, exactly. So you're going to have one of these for each degree of freedom.

Let me just point out a couple of elements here, and then I'll show you a demonstration because we have to have some fun today. These are point of informations. They're ratios, not absolute magnitudes.

That's number one. The second is-- I already told you, they got the same number. OK, each natural mode is associated with a particular natural frequency. This one goes with that one. This one goes with that one.

And once again, they're associated with-- yeah, let me say that. I need another board. Let's just go over here. So in a sense-- this is decouple. Decouple essentially into independent subsystems.

So in general, what the system's response looks like is-- I'm talking about the one in front of us here. This special case, where the masses and springs are equal. I think there's a minus sign in here.

Does this come up? Wonderful. That's it. Does everybody see that? Yes, sir.

AUDIENCE: [INAUDIBLE].

PROFESSOR: I'm sorry?

AUDIENCE: What does it say under that first bullet point?

PROFESSOR: Here?

AUDIENCE: These describe the situation.

PROFESSOR: These describe the situation in which the entire system is oscillating at. It's the second bullet here. Thank you.

AUDIENCE: At what?

PROFESSOR: At one frequency. Sorry, I'm just getting a little tired of writing. So, any other questions, problems, complaints? All right.

AUDIENCE: I have a question.

PROFESSOR: Yes, sir.

AUDIENCE: [INAUDIBLE]

PROFESSOR: That's correct. Then, let's see. Then there's a mistake right here. Thank you.

Yeah, because that's the way it came out. When you plug omega 2 having this value into here, the amplitude ratio comes out minus. Fair enough?

It threw me there for a minute. I thought you were going to say, why is the minus sign is there, rather than you could have had minus 1.618 and plus 1. And the answer is no reason, because these are ratios. Yeah, Kaitlin?

AUDIENCE: But shouldn't-- when we go back and look at what you wrote down, it's [INAUDIBLE]. I don't understand how that [INAUDIBLE].

PROFESSOR: I'm sorry, say again?

AUDIENCE: Never mind.

PROFESSOR: Find it? Yeah, they're ratios, It's just as simple as that. So you multiply them by any number and it still works. I'll actually show you here in a second.

At least, I believe that's the case. We'll just see here in a second. OK, questions? Comments?

All right. Now is the time. Could I bring up the side board here?

Let me show you-- anybody here taken 2086? Wonderful. I've got one person? Great.

Anyway, I believe in 2086, don't they teach you MATLAB? Isn't that the weapon of choice? OK, that's the program I'm using here, MATLAB.

For those of you who haven't seen it yet, it is definitely a mixed bag. I don't know how you feel about it. It's very-- yeah-- it's very powerful.

It stands for Matrix Laboratory. It was written, I don't know, 20, 30 years ago here, I believe, at MIT by people who were into matrices, into matrix algebra. And it's kind of command line oriented. The good news, it's very powerful. Whatever you want to do, you can do in MATLAB.

The bad news is, the user interface stinks. The language is very difficult to learn. It's even harder to remember. So with that rousing endorsement, let me show you what we've got here.

This is a program I've-- is that font readable by you guys? No? No?

AUDIENCE: [INAUDIBLE].

PROFESSOR: I'm sorry?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yes. Well, I believe I can-- here we go, fonts. Upping the fonts is kind of a mixed bag, because you get bigger letters but they're. OK, how's that?

So here's the situation. This is a MATLAB program. And I'll explain to you what it does as we go. Let me see if my little cursor-- my cursor's here, but I can't see it. All right, here's the system parameters.

Once again, we're doing a simple spring mass-- this simplified spring mass system, exactly the one we've done here. When I wrote it, you'll see I generalized it to do this guy. So we got M1 and M2, K1 and K2. But if you'll notice, you see here their values are equal.

We've got the mass at one kilogram each. And we've got 10 newtons per meter on each of the springs. Everybody appreciate that this system's numbers that we're putting in here match our case here, K over M? OK.

And you can see here, we've defined-- and again, let me just say I'm not trying to sell you on MATLAB. I don't want to leave you with the impression that we expect you to be able to instantly become a user of MATLAB. This is simply to illustrate the point of the lecture here.

Here is the M, the system matrix. There's the K matrix. And I'll show you the eigenvalue and eigenvector thing later. But let me-- take my word for it. See this here?

Ode45 is a cryptic allusion to the Runge-Kutta algorithm, fourth order Runge-Kutta that is the workhorse for integrating differential equations. And so let me just run this. And what I've got here is, here's the point I wanted you to get here, because I'll bet you can't see that cursor either.

Yes, anyway, see this right here? tspan is the time scale and the time step, defined up here. But these are basically the initial conditions. See it here?

X1, X1 dot, X2, X2 dot. So here's the first one. This is a 0.618 is for the X1. And 1 is for X2. Everybody appreciate that?

Got it? OK. If these are the initial conditions, what I've done, I have artfully chosen the initial conditions to have the same ratio.

What do you expect is going to happen? When I turn this thing-- I've got a simulation here. I'm going to run this, and you're actually going to see it. What do you think you're going to see?

It's a two spring, two mass system. What I've done is I've displaced the two masses.

AUDIENCE: [INAUDIBLE]

PROFESSOR: They'll certainly have an amplitude, because I'm putting it in there. That's the initial condition. The question is, what frequency you think they'll oscillate at?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Pardon?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Each of them will oscillate with the same frequency, for sure, but what do you think it's going to be?

AUDIENCE: That one.

PROFESSOR: It's going to be that one. So off we go. So let us hope that yours truly's program worked. Here we go.

Oh, look at that. [INAUDIBLE], please interpret that for me. What do you see there? Hang on a second.

Let me blow it up so you can see it. Ooh, isn't that pretty? And I believe the blue is X1 and the green is X2.

See? Everybody agree? Everyone appreciate what's going on?

You pull them both at slightly different-- you basically used the first natural mode as the initial condition. And sure enough, they oscillate together. They oscillate at the same frequency. They oscillate at that frequency.

Let me just see-- I just want to make sure we get the full value out of this thing. Well, of course you can't see it anymore because our numbers are so big. Well, that's great. Anyway, take my word for it at-- oh, here it is.

The period for the first natural frequency-- or I guess it's the second-- it should be like 1.2. Or is it 3? Yeah, I'm sorry, the period is 3.2. And sure enough, there it is.

It's about 3. 3.2. Fabulous. Everybody got it?

OK, now watch closely. Let me see if I can do this. This requires a little dexterity, which is always a short supply here. I have to hit this and this.

Make sense? That's what it actually looks like. They're both oscillating at the same natural frequency, going up and down together. But they have different amplitudes. So one's bigger than the other.

So that's what it looks like. Questions? Christina, you good? Clear enough? Wonderful.

So let's go to our program. And instead of that set of initial conditions, we'll do the other. Read them to us here. What are the initial conditions here?

AUDIENCE: It's 1.618.

PROFESSOR: That's right, it's this guy.

AUDIENCE: That guy.

PROFESSOR: It's this guy. It's this ratio. So I basically arbitrarily chose, is it the negative first? No.

I chose that one over there, 1.618. And then a minus 1 for the second one. Fair enough?

OK, there it goes. We've got to save it and make sure we got it. So again, you got a clue what's going to happen here?

Here we go. Boom. Look at that. What's going on there? Yikes.

Explain me. Is that good, bad, indifferent? Is it right? Wrong?

AUDIENCE: The way the system acts, it has a higher frequency.

PROFESSOR: Yeah, exactly. Two things. One is, they're out of phase.

They're doing this. One's going this way, and the other's going the other at different amplitudes but the same frequency. But the frequency in question is higher than the previous.

AUDIENCE: Why are they out of phase?

PROFESSOR: I'm sorry?

AUDIENCE: Why are they opposite of each other?

PROFESSOR: [INAUDIBLE], why are they opposite each other? Because we made them that way. We said, that's the initial condition. Does that make sense?

That minus sign does. One starts out, and one starts in. And they do that. Clear enough?

Now what's going to happen if we plain just choose any old initial condition? These were special. We worked like a dog to compute these, so that the system would decouple in that way. So what if we-- now let me put that back.

Now look at this one. All right, look at that. Read that to me.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah. So that says the initial condition for the first mass is 1 and whatever that is. One whatever that is. The second masses' initial condition is half that in the same direction.

Both positive. So Christina, they're going to go together. But [INAUDIBLE], at what frequency? Any idea what it's going to look like?

If you do, you're a better man than I, because what you're going to see here is that. It's this thing right here. It's that expression right there.

And here's what it looks like. Did I stop the-- oh, wait a minute. Did I ever show you that before? I think I forgot to show you the other.

Anyway, not to worry. Hang on a second. I've got to stop this guy. First I have to find my finger.

There it goes. That's the previous case. When they're out of phase, different magnitudes, going in opposite directions. And you can see, they're going at a higher frequency than before. Make sense?

So now we are-- just to refresh your memory-- now we're going for the third case, in which there's nothing special. We just picked a couple of initial conditions out of a hat. And here we go.

Oops, I think not. I think that's the previous case. So let's go here. This is another wonderful thing about MATLAB is nothing happens until you save it. So we were just running the previous case. Nasty. Look at this.

All right, can everybody see that? If you can interpret this, you're smarter than I am. But what this is, this is simply this expression over here. It's this expression for just some arbitrary initial condition.

Do you see that that behavior though? Each of them, they're going together kind of, but they-- anyway, watch this. Here's what the simulation of that looks like.

What the heck is that? Well anyway, the point of the story is that multiple degrees of freedom system in general's response can be arbitrarily complicated. It's not arbitrarily complicated, but pretty complicated.

You'll get, in general if it's an nth order system, if you don't know anything about the worst case, you'll see four frequencies in there. And they're all mixed together in some mystical way that's unknown to you. Fair enough? And it's only when you reach the natural modes that you actually find out what is going on here.

Well now I have to turn your attention to this guy. This is made by Professor Vandiver's machinist, a perfect example of a second order system. And I bring it to your attention here for two-- at the end of the day what we're going to do is I'm going to demonstrate exactly what I just did for the textbook case, the textbook system. I want to demonstrate exactly the same thing for this guy, only this is a real system.

Very nice. We have a steel rod. It must be a half inch in diameter. The whole thing weighs several pounds. These sliding masses are right circular cylinders with a hole drilled through them.

It's ever so slightly larger than these here. They're of different lengths. They're made of brass. They're serious masses.

And the springs, which extend from here to here, and from here to here are wound on a lathe, and attached, and so forth. Pretty, no? Now look right off the bat. Did you see how that thing operates?

Would you agree you have some complicated behavior here? Now also would you agree that this is it like that? Everybody see that?

Before we go too far, this is a mixed message here. [INAUDIBLE], is this exactly like that? In what way is it similar to that?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Well, what is clear is that you've got two springs and you've got two masses. About that there is very little argument.

AUDIENCE: It's damped.

PROFESSOR: It's damped. Can everybody see that? How does [INAUDIBLE] know that it's damped? How's he know it's damped? I mean, that's just a wild guess on his part, but.

AUDIENCE: You can hear it, and it slows down. And it slows down. This is the most important part is that it stops. Eventually if you come back in a minute or two, it's done.

PROFESSOR: All right, [INAUDIBLE]. You're on a roll. There's definitely damping there. What kind of damping?

AUDIENCE: Friction.

PROFESSOR: Friction, yeah. Does that have another name that you can think of? It's definitely friction.

What it's not is viscous friction. What it is not is a damper or a dashpot which we've shown you before with the ideal expression that generate a force that opposes the-- generation of an opposing force that's linearly proportional to the velocity. What's going on here, do you think?

What kind of damping do you think? It's called Coulomb. This is called Coulomb damping.

And this is a digression. Now we're on the part where this is really-- everything that's on the board is what I wanted you to really come away from today with. So now we're out kind of in the, I would call it the winging it area right here, because this is the part where I simply had fun with the demo.

This is viscous. And this has got the symbol-- well anyway, this is what it looks like. And this is the force of the damper. We'll call it B.

And this is the velocity. And this is for constant of proportionality B. And it has this little symbol, like that.

And when equations of motion are solved that contain that, the response looks like this. What we're talking about here, the force put out is a constant. That just comes from the sliding.

And what it generates are distinctly non-linear equations of motion. And what you get here is you get this kind of behavior. If you really looked at it, what you'll see is there's definitely damping for large motions when the inertial forces and so forth are large compared to the friction forces. It'll look a lot like conventional viscous damping. It's just that when motions get really small, and the forces get down there to on the order of this, all a sudden you'll see on one cycle it'll just stop.

And were you up here where you could see, or if we had a closeup of this-- you can't see it, but just watch this thing stop. Right there. Do you see that?

That's a little hard for you to see from there, but watch. Anyway, were you up here, you'd see this. That's what we're looking at.

Well here we go. I need some help here. Who's in a volunteering frame of mind?

Amy, all right. I appreciate the help here.

Here's what I want to do. We just blew out some wonderful theory. All this is just solid as a rock. Yes, sir.

AUDIENCE: For the Coulomb friction, is that a linear [INAUDIBLE]? Or is it still exponential?

PROFESSOR: I'm sorry? Oh, no. If I'm not mistaken, I didn't really look this up, but I believe it's linear. I'd have to-- take that with a grain of salt, but I believe it's linear.

Yes, Amy, here's the situation. We have all this marvelous theory. My goal is to-- and we have this fabulous demo apparatus, though inherited. And what I'd like to do-- oh, and we have computational means to.

And in fact, we just went through the exercise. We already know those same equations that work for this work for this. Those are general.

However, it's not my piece of apparatus. And well, here's the deal, what are the Ms and Ks. What are the values of-- I need M1, K1, M2, K2 to put into the.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, Yeah. That's what I'd like to do is I'd like to.

AUDIENCE: Do I have to just determine the it by looking at it?

PROFESSOR: Oh no, no, no. No, no.

AUDIENCE: I'm not that good.

PROFESSOR: You're my assistant.

I guess the question is, how would you-- and I've got to tell you, that's the math part of a program. Now we're in the engineering part of the program, because somebody gave you a real live demo apparatus. Works like crazy, or appears to. And I'd love to take advantage of it, but I don't know any of the numbers.

AUDIENCE: [INAUDIBLE]

PROFESSOR: No. That's the constraint I'm operating on. It doesn't belong to me. I mean, I could take it apart. That's an absolutely appropriate thing to do.

I would have liked to. It would be easier if you could. You just go, take a screwdriver to it.

Here, put this on there and pull this out. I didn't have the luxury of any of that, so what's your next best suggestion? Nice suggestion, but no cigar. I'm sorry?

AUDIENCE: Take it apart anyway, put it back together before the person notices.

PROFESSOR: Well yeah. Yeah, no. That's fudging. Yeah, they notice.

Have you ever taken apart anything made in modern manufacturing method? Oh, it's good because you can't put them back together. They're assembled by machine.

And once upon a time you could disassemble one and reassemble things without detection. But anymore, once you take them apart, it's wicked hard to get them back together. OK, the floor is open. I need another suggestion. What are you going to do?

AUDIENCE: Do you need to know the exact K and M, or do you just [INAUDIBLE] another ratio?

PROFESSOR: I thought you were going to-- I need to know M or K. I need to know them all.

AUDIENCE: Do you know the density of the--

PROFESSOR: I was going to say, but I don't need to know anything exactly. All I need to know is as good a guess as you can come up with. It's all an estimate.

AUDIENCE: If you know the density of the material, you can easily work up [INAUDIBLE]. I'm assuming you're about [INAUDIBLE].

PROFESSOR: Oh absolutely, absolutely.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Wonderful. Absolutely. She hit the jackpot, rang the magic buzzer.

That's exactly what I did. Here's a little crummy sketch. Oh wait, you can't see that. Anyway, these are right circular cylinders with holes in them. And they've got measurements beside them.

I can tell you, this is 75 millimeters. This is 35 millimeters. This one is 37 millimeters long. So I did that.

That's great. That's an excellent suggestion. And after I did exactly that, I won't write out the formula. You know area, and volume, and all of that. Let me get you the right order here.

M1 is 0.2929, and M2 is 0.5938. Everybody got that? This was obtained by taking a ruler to these things, taking diameters, lengths, and diameters of holes, multiply them times the density of brass taken out of the book.

Do you believe that? Do you believe that number? Well, you're a trusting soul. I don't. To me, I believe that number.

This was done with a ruler. The little millimeter thingies. So I just say, don't fall in the trap of false precision.

OK Amy, you're on a roll. We've got the masses. What now?

AUDIENCE: Free body diagram.

PROFESSOR: Yeah, we got all that.

AUDIENCE: Yeah, you've got that. But then what you can do for the spring, the forces of the spring when static. Don't move it. Don't move it. So take the top mast. It's not moving, which means that you know that the force going upwards-- which is the spring-- is equal to the force going downward, which is [INAUDIBLE]. So you can measure the displacement from the start of the spring to the bottom of the spring.

Do you know the natural length of the spring?

PROFESSOR: No Anyway, what I was going to say is, excellent idea. Can't do it.

But what Amy was basically saying is, you know the masses now. Why not simply take from that expression right there, MH over K, right? What's the problem with that? How about it? Devin, how come I can't do that?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, I really don't know the no load position. Is that it, Amy? Right there? Maybe.

Devin, you're the one that suggested it. Are those the no load positions of the masses? And if not, why not? I did mean to give you a clue there. Yeah, Nick?

AUDIENCE: It can't be because there's static friction.

PROFESSOR: Exactly. So Nick, you brought it up. What's that number? You don't know that either. No.

Like I said, Amy, nice idea. No cigar. What else? We're running out of time? Here we go, Douglas.

AUDIENCE: Could you displace each mast a certain [INAUDIBLE], and then measure the time it takes for them to stop and get the damping ratio?

PROFESSOR: Hit the damping ratio. Well, I'll tell you what-- you want to say that again? He said displace one or both count oscillations and get the damping ratio. Nick?

AUDIENCE: Do we have anything like a force gauge or a spring scale?

PROFESSOR: No. This is my living room I'm talking-- or my study. Anyway, Douglas said-- I forgot what you said now.

He said-- oh, I know. You said, displace it and count the oscillations. Get the damping ratio.

First off, the damping ratio is no help, even if we did get it. And the only formula for which we've ever given you-- the only formula we've ever given you to do that with pertains to this kind of friction, which is not present. No cigar.

Nick says, how about force gauge? Now, don't have it.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, we are.

AUDIENCE: [INAUDIBLE] displace the other one and then find the frequency.

PROFESSOR: Oh, what's your name? Sean? Or John?

AUDIENCE: Sean.

PROFESSOR: Say that out loud. Say it loud enough that Devin can hear you.

AUDIENCE: You hold the first mass, and then you displace the second one.

PROFESSOR: Hang on. He says, hold the first mass like this set screw right here. And?

AUDIENCE: And then displace the other one then.

PROFESSOR: Like that?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Actually, to answer your question Nick, the only instrument I have is a clock. Hang on a second. Here it is. Of course, this is the big task is finding it.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Count to 10, remember like Vandiver told you. Skip 1, 1, 2, 3, count to 10. Stop. Excellent, excellent, excellent.

When you do that, that's the second one. I did that and right here, right here it's TP. The period of 10 of them-- and then I divide to get 1-- is 0.83 seconds. What does that tell you, Sean?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Well actually, these two-- you can get the frequency. But what this does, because this gives you the frequency, you know in general that-- in particular, you know that this second natural frequency, which is just associated with this single spring and a mass here. It's just this guy. It's not both of them.

Anyway, this turns out to-- I didn't graph that. That's square root of K over M. Trust me, you can put those two together, and you get K2 is equal to-- newton meters. This is exactly what you said.

Freeze the first mass. Displace the second. Measure the period. You get the natural frequency for basically K2 over M2. So you got K2 out of it. Yeah, Douglas?

AUDIENCE: So how come it gives you just the natural frequency and a damp natural frequency?

PROFESSOR: Oh no. It absolutely is. It's all damped, no question about it. But again, what we're doing is we're going close enough, right? Because I have nothing.

So even the damped natural frequency is better than nothing. All right, so Amy back to you. You're back in business. What now? So now we've got M2 and K2.

AUDIENCE: You just need K1.

PROFESSOR: Yeah. Now we need K1. What do we do now?

AUDIENCE: We want to do the same thing that we just did for K2. [INAUDIBLE].

PROFESSOR: Exactly. Well, not quite. Let's see, now I'm going to turn loose-- now we're back to our original system.

It doesn't hurt anything. It's just ugly to look at. Now what? Sean, do that trick again.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Push this one up?

AUDIENCE: [INAUDIBLE]

PROFESSOR: I don't think so. Yeah, Nick.

AUDIENCE: So just fix the second mass.

PROFESSOR: Fix the second mass.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah, you see this frequency here? I'm going to overrule Chandler. I'm going to say, this is the natural frequency of that's mass and these two springs.

Do that same trick again. You get the equivalent spring rate, subtract the second from it, and you get the other one. Devin, is that what you were going to say? Wonderful.

That is exactly what I did. And you get out of it, you get K1 is equal to 50.45 newton per meter. In the interest of time, I'm going to short circuit this. I took exactly these parameters, I put them into that same computer program we had before, and what came out-- I have to have a place to put it.

Ah, wonderful. And now I have to find it. Here we go. Here are the two modes. Actually, let me put it right here.

For this system, because here's the first one. And here's the second one. 0.9760 and minus 0.2177. Everybody appreciate that? This is by the same computational procedure we spoke of earlier.

And sparing no expense, we have here a, made fresh from my basement, a custom made initial condition setting device, which I can hopefully avoid killing myself with. OK here's what we have.

So I guess I didn't show you this first. If you can see it, what we have marked is the reference position. That's the rest position that we couldn't find by laying it down. Or excuse me, this is the static equilibrium position of mass number one, static equilibrium position of mass number two.

Mode one or there. What they said over there, 0.4. And .97 is down here. Mode two is over here.

So Devin, while I'm doing this, tell me how am I going to know if this is right or if this is all just bogus? What observable's going to tell me that I got it right?

AUDIENCE: [INAUDIBLE]

PROFESSOR: I'm sorry, speak up.

AUDIENCE: [INAUDIBLE]

PROFESSOR: How about it, Nick?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Exactly. And you ought to be able to see it from where you are. Can you appreciate that they're not at the moment? All right, now hang on.

Here comes mode number one. This takes two hands to do it. All right, you ready?

This is mode number one. Again, it's a those numbers. How about it? Can you see it? Very good.

How about the other? And so here, this is number two. And this is a little more complicated, because the other one has to be done from the bottom. Hang on a second.

Now this one we're doing is we're deflecting-- this one is positive downward. So X1 is down, but X2 is negative. So it's displaced upward a bit. Are you ready? Nick, what do you expect to see this time?

AUDIENCE: The frequency should be higher and they'll move in opposite directions.

PROFESSOR: That's the key. Once again, they're going to move with the same frequency, albeit in different directions. But that new frequency is going to be higher than before. And sure enough, stand back.

So there you go. What did that tell us? That told us that the first order we got the system parameters identified correctly and the theory holds up.

Questions? Comments? Complaints? Devin?

AUDIENCE: [INAUDIBLE]

PROFESSOR: I'm sorry?

AUDIENCE: What was the second set of conditions?

PROFESSOR: The second set of initial conditions were right here. This is the second mode, X1, 0.97, X2, minus 0.2. OK, have a great Thanksgiving holiday.