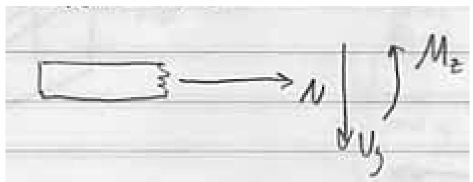
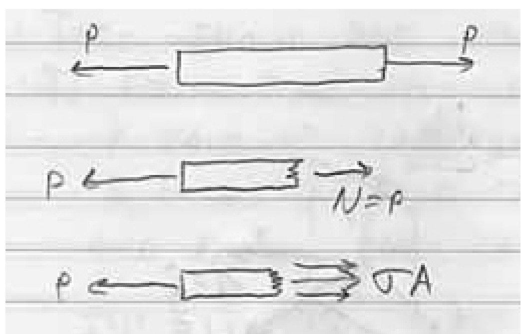
2.001 - MECHANICS AND MATERIALS I
Lecture #12
10/23/2006
Prof. Carol Livermore

MULTI-AXIAL STRESS AND STRAIN

Recall: Internal Forces and Moments



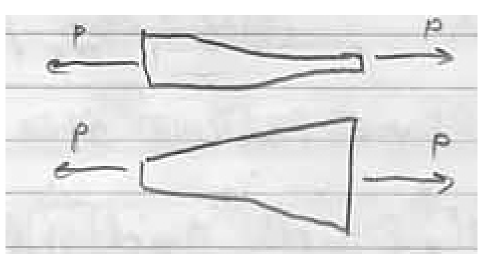
Axial stress from uniaxial loading



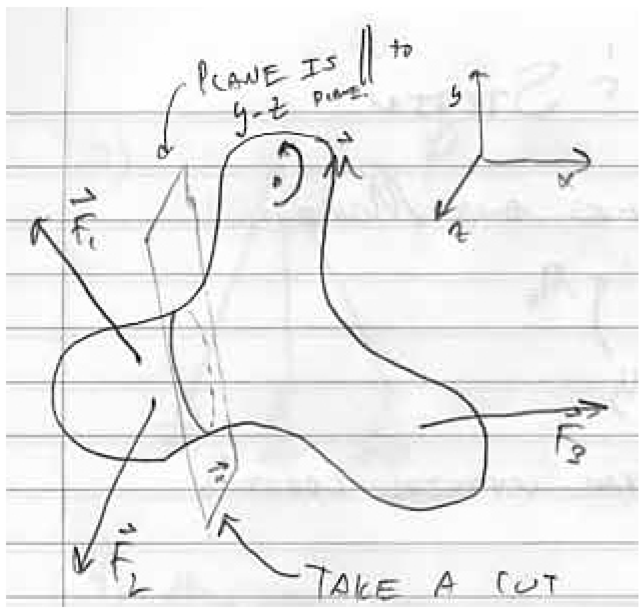
$$\sigma = \frac{P}{A}$$

Note: σ is an average axial stress.

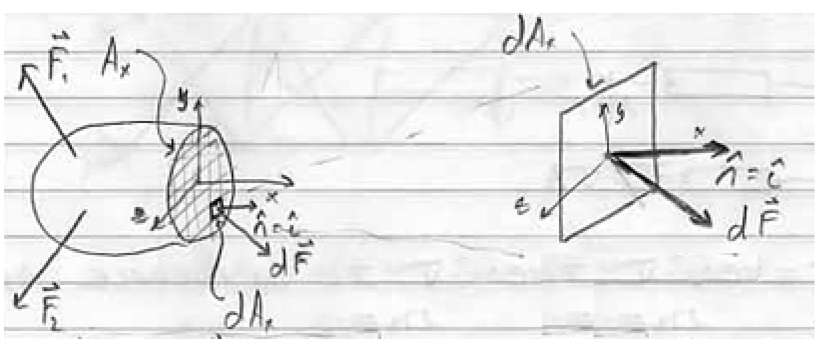
For slender (long, thin) objects A is either *uniform* or *slowly-varying*.



What about a more general case?



Note: $d\vec{F}$ is *not* in general parallel to $\hat{n}$.



$$d\vec{F}^{(i)} = dF_x^{(i)}\hat{\imath} + dFyx^{(i)}\hat{\jmath} + dF_z^{(i)}\hat{k}$$

$$\hat{n} = \hat{\imath}$$

$$N_{totalforce} = \int_{A_x} dF_x^{(i)}$$

$$\bar{\sigma} = N_{totalforce}/A_x$$

Traction: $\vec{t}$ Force per unit area at a point

$$\Rightarrow \quad \vec{t}^{(i)} = \frac{dF^{(i)}}{dA_x} = \frac{dF_x^{(i)}}{dA_x}\hat{\imath} + \frac{dF_y^{(i)}}{dA_x}\hat{\jmath} + \frac{dF_z^{(i)}}{dA_x}\hat{k}$$

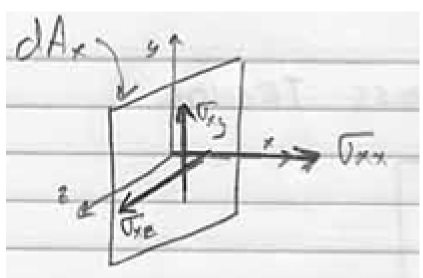
$$\sigma_x x = \frac{dF_x^{(i)}}{dA_x} \text{ (Normal Stress)}$$

$$\sigma_x y = \frac{dF_y^{(i)}}{dA_x} \text{ (Shear Stress)}$$

$$\sigma_x z = \frac{dF_z^{(i)}}{dA_x} \text{ (Shear Stress)}$$

So:

$$\vec{t}^{(i)} = \sigma_{xx}\hat{\imath} + \sigma_{xy}\hat{\jmath} + \sigma_{xz}\hat{k}$$



$$N_{TOTALFACE} = \int_{A_x} \sigma_{xx} dA_x$$

$$V_{TOTALFACE_y} = \int_{A_x} \sigma_{xu} dA_x$$

$$N_{TOTALFACE_z} = \int_{A_x} \sigma_{xz} dA_x$$

To find the stresses on the
y face $\Rightarrow$ take cut on x-z plane.
z face $\Rightarrow$ take cut on x-y plane.
And follow the same procedure.

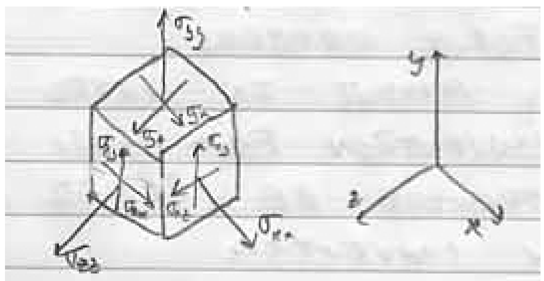
$$\vec{t}^{(j)} = \frac{dF_x^{(j)}}{dA_y}\hat{\imath} + \frac{dF_y^{(j)}}{dA_y}\hat{\jmath} + \frac{dF_z^{(j)}}{dA_y}\hat{k}$$

$$\vec{t}^{(j)} = \sigma_{yx}\hat{\imath} + \sigma_{yy}\hat{\jmath} + \sigma_{yz}\hat{k}$$

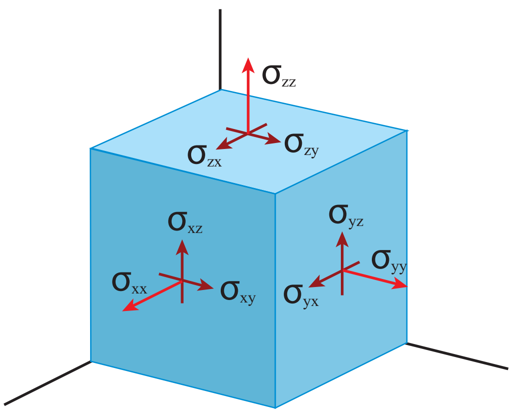
$$\vec{t}^{(k)} = \frac{dF_x^{(k)}}{dA_z}\hat{\imath} + \frac{dF_y^{(k)}}{dA_z}\hat{\jmath} + \frac{dF_z^{(k)}}{dA_z}\hat{k}$$

$$\vec{t}^{(j)} = \sigma_{zx}\hat{\imath} + \sigma_{zy}\hat{\jmath} + \sigma_{zz}\hat{k}$$

Materal Point



See handout for more clear diagram.



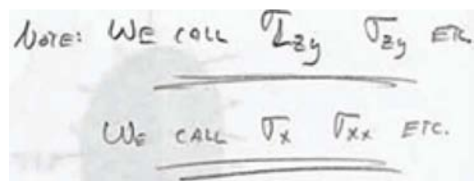


Figure by MIT OCW.

Matrix form of stress tensor

$$[\sigma\ ] = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix}.$$

Diagonal terms are normal stresses.
Off diagonal terms are shear stresses.