Mass Conservation

Conservation of mass means mass is always conserved, it does not appear or disappear.

If we look at a pump from a qualitative point of view, mass conservation says that:

Rate mass goes in = Rate mass goes out + Rate mass accumulates in pump

Quantitatively, this is expressed as:

$$\dot{m}_{In} = \dot{m}_{Out} + \frac{\partial\, m_{Fluid\ In\ Pump}}{\partial\, t} \qquad \textbf{\textit{Equation (1)}}$$

Rate at Which Mass Flows In

Rate at Which Mass Flows Out

Rate of Increase of Mass Inside of The Pump

If we can assume:

1. <u>Rigid Pump</u>: The pump body can't change shape, meaning $\textbf{Volume}_{\textbf{PUMP}}$ is constant.

2. <u>Incompressible</u>: The fluid in the pump can not be compressed into a smaller volume so the amount of fluid per unit volume (density = ρ_{FLUID}) is constant.

3. <u>Full Pump</u>: The pump remains full of fluid (not draining or filling) during operation.

Then we can say that the mass inside of the pump remains constant, or:

$$\frac{\partial\, m_{Fluid\ In\ Pump}}{\partial\, t} = 0$$

To help you see this, think of a pump as a full cup of water. If the cup does not change shape (i.e. assumption 1), the water is not compressible (i.e. assumption 2) and the cup was initially full (i.e. assumption 3), then the water you pour into the cup must equal the water that comes out (over flows from) the cup. So we can say that the rate at which mass in the pump/motor changes is negligible.

$$\dot{m}_{In} = \dot{m}_{Out} + \cancel{\frac{\partial\, m_{Fluid\ In\ Pump}}{\partial\, t}}$$

Rate at Which Mass Flows In
[Pour water into cup]

Rate at Which Mass Flows Out
[Water overflows]

~ 0

Rate at Which Mass INSIDE Changes
[Mass inside of cup stays the same]

Mass Conservation Cont.

This leaves us with;

$$\dot{m}_{In} = \dot{m}_{Out} \qquad \textit{Equation (2)}$$

Rate at Which Mass Flows In

Rate at Which Mass Flows Out

Intuitively, this makes sense because if you can not add to, or subtract from something, then what goes in must equal what comes out.

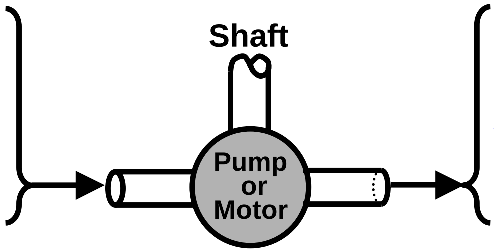
Now we need to put this in terms which are important to pumps and motors.

FLOW IN:

A_{In}

$\overline{V}_{In}$

ρ_{In}



FLOW OUT:

A_{Out}

$\overline{V}_{Out}$

ρ_{Out}

WHERE:

Ai = Cross Sectional Area of Port i

Vi = Average Velocity at Port i

ρι = Fluid Density at Port i

Important Flow Variables In Hydraulic Pumps and Motors

The mass flow in a pipe or pump/motor entry or exit (the inlet or outlet ports) is:

$$\dot{m}_k = \rho_k \bullet A_k \bullet \overline{V}_k \qquad \textit{Equation (3)}$$

Where k characterizes the mass flow as in or out of the pump. If we substitute equation 3 into equation 2 we obtain:

$$\rho_{In} \bullet A_{In} \bullet \overline{V}_{In} = \rho_{Out} \bullet A_{Out} \bullet \overline{V}_{Out} \qquad \textit{Equation (4)}$$

Since we are assuming the fluid is incompressible, the density of the fluid will remain constant and $\boldsymbol{\rho_{In} = \rho_{Out}}$. In equation 4, the densities cancel out leaving:

$$A_{In} \bullet \overline{V}_{In} = A_{Out} \bullet \overline{V}_{Out} \qquad \textit{Equation (5)}$$

The quantity A x V is also know as the volume flow rate, Q. Therefore equation 5 can be written as:

$$Q_{In} = Q_{Out} \qquad \textit{Equation (6)}$$

Equation 6 is the result of mass conservation which is useful to hydraulic systems analysis