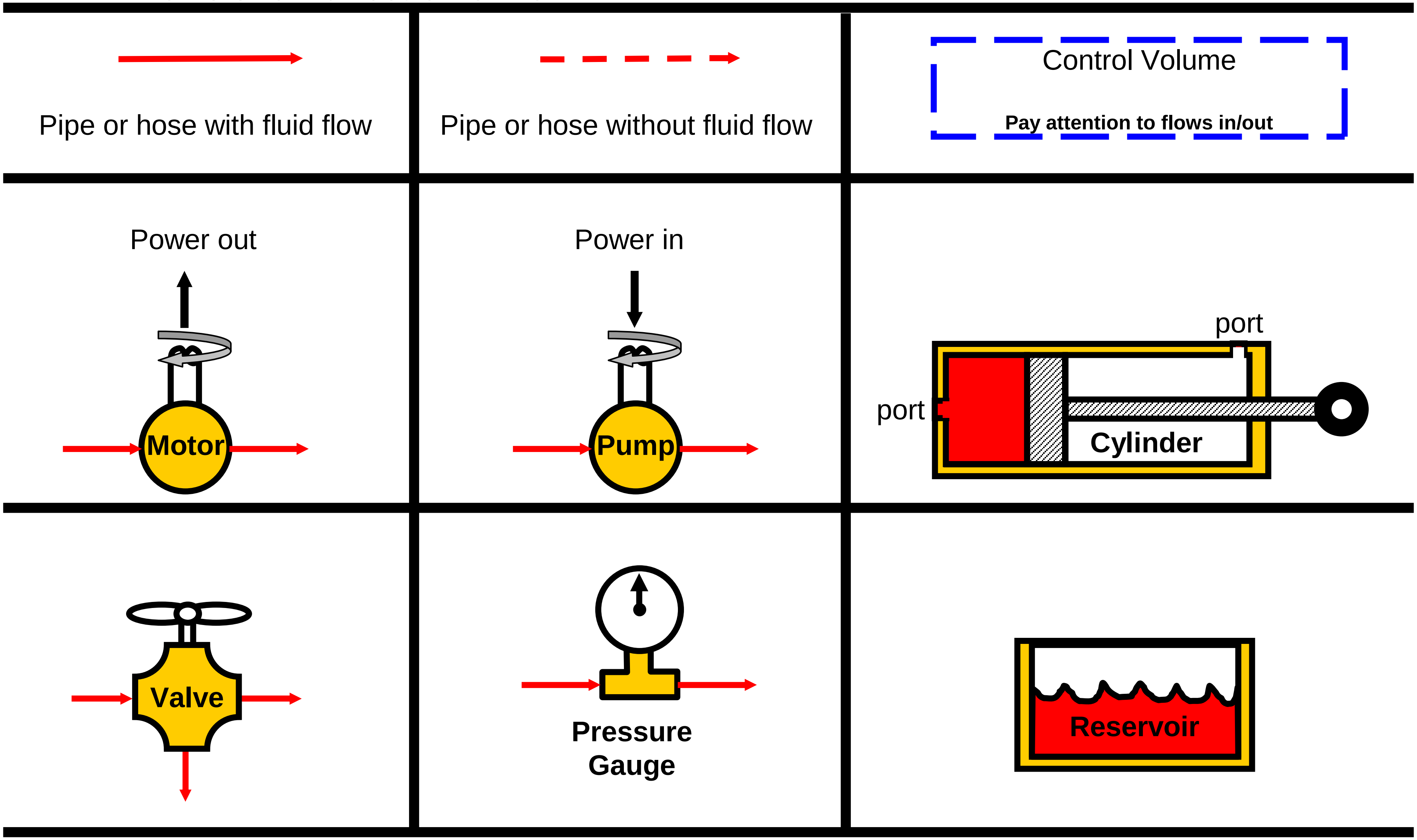
# LECTURE 8

## Hydraulic machines and systems II

# Basic hydraulic machines & components

## Graphical Nomenclature

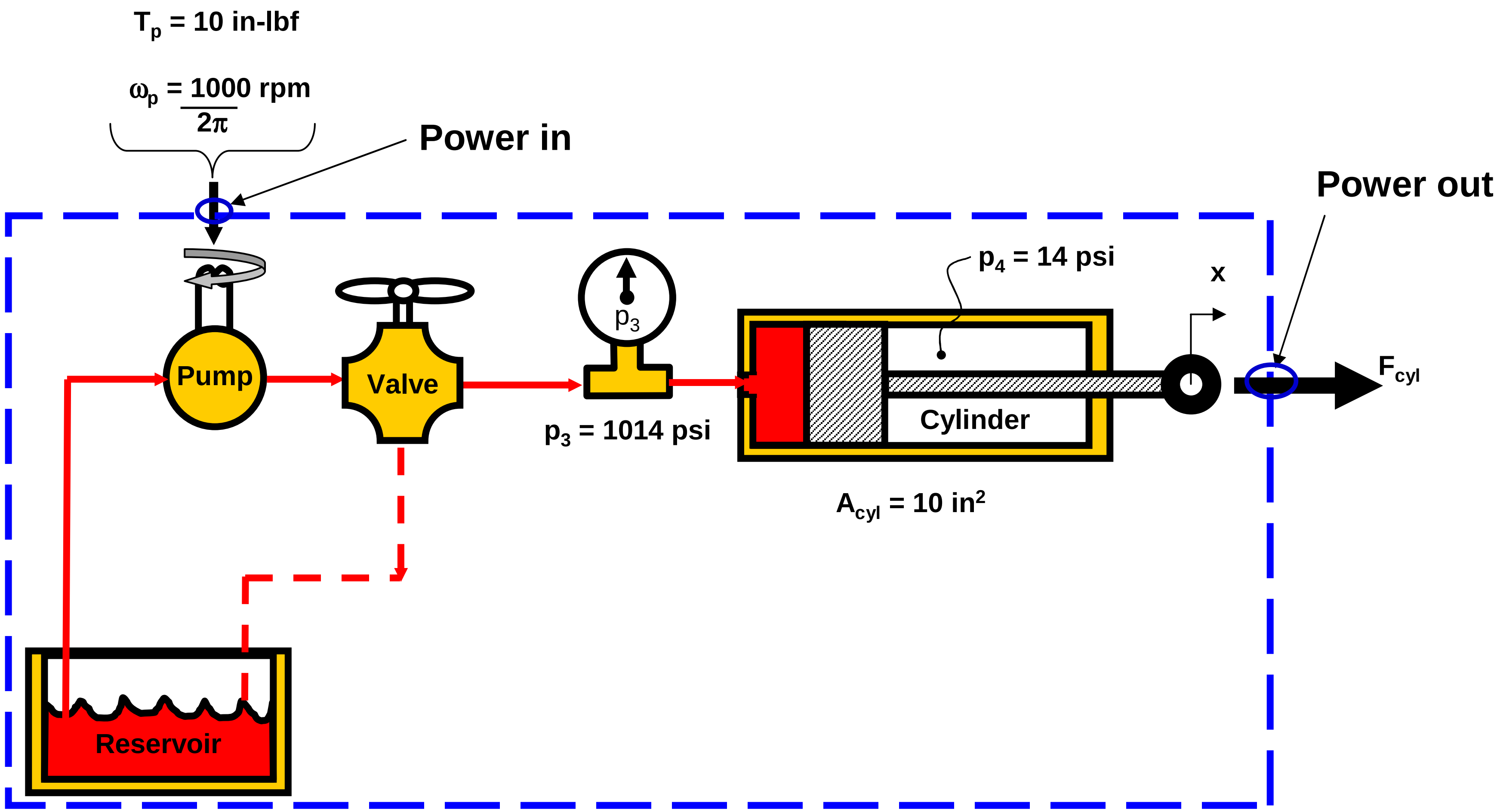
- Arrows show direction of flow



# Example I – Pump & cylinder

**Solve for the the velocity of piston and the force exerted by piston**

**Note where power crosses into and out of the system boundary**



# Example I – Pump & cylinder cont.

## Force exerted by piston:

- $F_{cyl} = A_{cyl} \Delta p_{cyl}$
- $\Delta p_{cyl} = p_3 - p_4$ = 1014 psi – 14 psi = 1000 psi
- $F_{cyl} = 10\ in^2 \ \ 1000\ {}^{lbf}/_{in^2}$ = 10 000 lbf

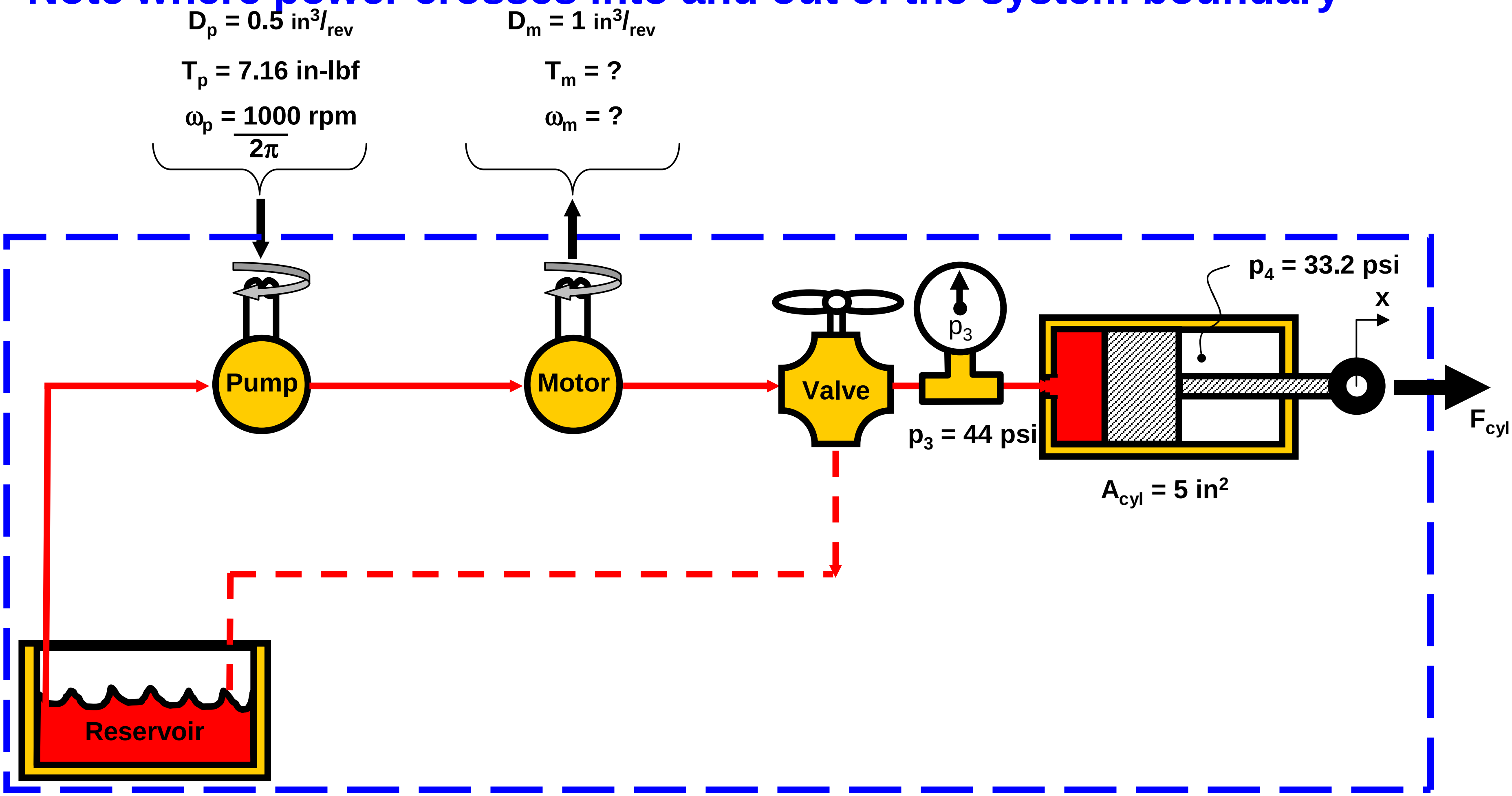
## If we know F and v, we know the power output of the cylinder

- **At the boundary of the hydraulic system we see one inflow & one outflow of power**
- **From the power balance:**
- $\Sigma P_{in} = \Sigma P_{out} + \Sigma P_{loss} + \Sigma (dE_{stored}/dt)$; **If** $P_{loss}$ **&** $(dE_{stored}/dt)$ **are small compared to** $P_{out}$:
  - ✓ $\Sigma P_{in} \sim \Sigma P_{out}$
  - ✓ $T_p\ \omega_p \sim F_{cyl}\ v_{cyl}$
  - ✓ $v_{cyl} \sim (T_p\ \omega_p)/F_{cyl}$
    - $= (10\ \text{in-lbf})\ (1000/2\pi\ {}^{rev}/_{min})\ (2\pi\ {}^{rad}/_{rev})\ (1/60\ {}^{min}/_{s})\ /\ (10\ 000\ \text{lbf})$
    - $= 0.0167$ in/s
    - Always check and list your units!!!

# Example II – Pump, motor, & cylinder

**Given the diagram, solve for $T_m$ and $\omega_m$**

**Note where power crosses into and out of the system boundary**



# Example II – Pump, motor, & cylinder cont.

**Motor speed: We know that the mass flow rate through the pump and motor has to be the same. As we assume the liquid is incompressible, this means the volumetric flow rate is the same:**

- $Q_p \quad = \omega_p D_p = Q_m = \omega_m D_m$
- $\omega_m \quad = \omega_p (D_p/D_m)$

  $= [1000/(2\pi)\ ^{rev}/_{min}]\ [(0.5\ in^3/_{rev}) / (1\ in^3/_{rev})] \quad = 500/(2\pi)\ ^{rev}/_{min}$

**Motor torque:**

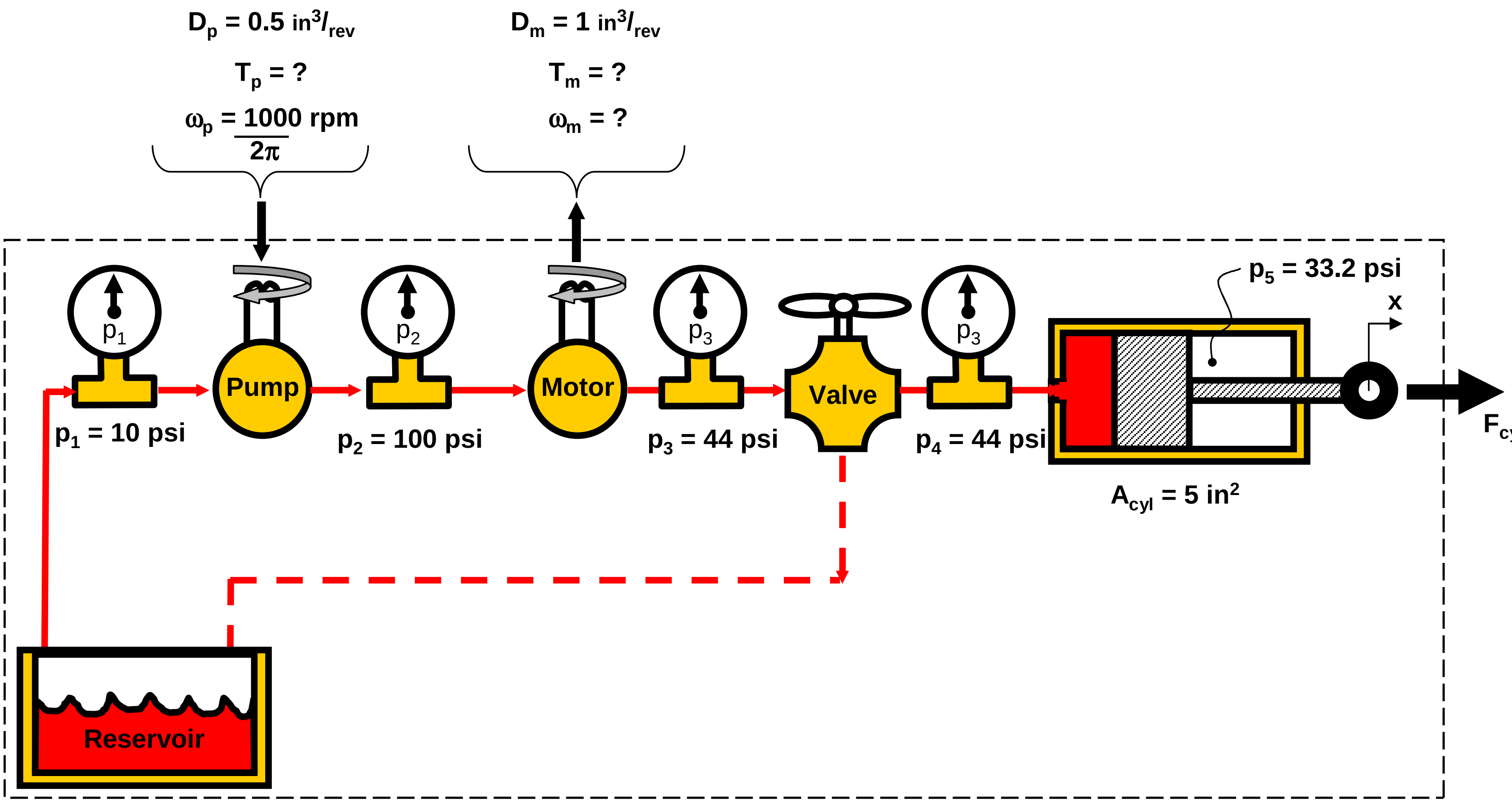
- $\Sigma P_{in} = \Sigma P_{out} + \Sigma P_{loss} + \Sigma (dE_{stored}/dt)$ ; If $P_{loss}$ & $dE_{stored}/dt$ are small compared to $P_{in}$:
  - ✓ $\Sigma P_{in} \sim \Sigma P_{out}$
  - ✓ $T_p \omega_p \sim T_m \omega_m + F_{cyl} v_{cyl} \sim T_m \omega_m + (\Delta p_{cyl} A_{cyl}) v_{cyl}$
  - ✓ $T_m \sim [T_p \omega_p - (\Delta p_{cyl} A_{cyl}) v_{cyl}] / \omega_m$
- We can not solve as we don't know $v_{cyl}$, we find $v_{cyl}$ via volumetric flow rate
  - ✓ $Volume_{cyl} = A_{cyl} x_{cyl}$ ; $Q_{cyl} = d(Volume_{cyl})/dt$; $Q_{cyl} = d(A_{cyl}x_{cyl})/dt = A_{cyl} v_{cyl}$
  - ✓ $Q_{cyl} = Q_p = Q_m$ therefore $\omega_m D_m = \omega_p D_p = A_{cyl} v_{cyl}$
  - ✓ $v_{cyl} = \omega_p D_p / A_{cyl}$
  - ✓ $T_m \sim [T_p \omega_p - (\Delta p_{cyl} A_{cyl}) v_{cyl}] / \omega_m \sim [T_p \omega_p - (\Delta p_{cyl} A_{cyl}) (\omega_p D_p / A_{cyl})] / \omega_m$

  - ✓ The "numerical plug and chug" is left to you, $T_m = 8.91$ in lbf

# Example III – Pump, motor, & cylinder cont.

**Given the diagram, solve for $T_p$, $T_m$, and $\omega_m$**

**Note where power crosses into and out of the system boundary**



# Example III – Pump, motor, & cylinder cont.

## Use a power balance <u>on the pump</u> to determine the pump torque:

- **$\Sigma P_{in} = \Sigma P_{out} + \Sigma P_{loss} + \Sigma (dE_{stored}/dt)$; If $P_{loss}$ & $(dE_{stored}/dt)$ are small compared to $P_{in}$:**

  $\Sigma P_{in} = \Sigma P_{out}$

  $T_p \, \omega_p = \Delta p_p \, Q_p$

  $T_p = \Delta p_p \, (Q_p) / \omega_p = \Delta p_p \, (D_p \, \omega_p) / \omega_p = \Delta p_p \, (D_p)$

  $(100\text{psi} - 10\text{ psi}) \quad 0.5 \, {}^{in^3}/_{rev} \, ({}^{1}/_{2\pi}) \, {}^{rev}/_{rad} = 7.16 \text{ in lbf}$

- **The solution to the rest of the problem is the solution to Example II**

# PROJECT I AND HWK 6

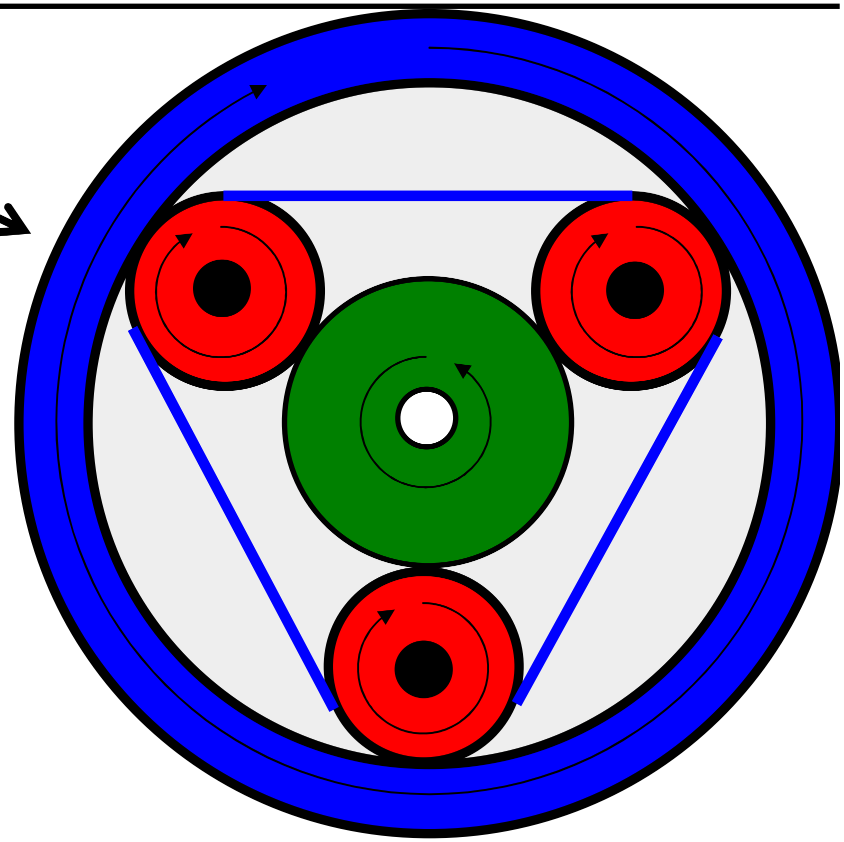
# PLANETARY GEAR TRAINS

# Planetary relationships (ala Patrick Petri)

## Say the arm is grounded….

- Planet gears = idler gears

$$\frac{\omega'_{ri}}{\omega'_{si}} = -\frac{N_s}{N_r}$$



## Now say the arm spins…. we can say

$$\omega'_{ri} = \omega_{ri} - \omega_a$$

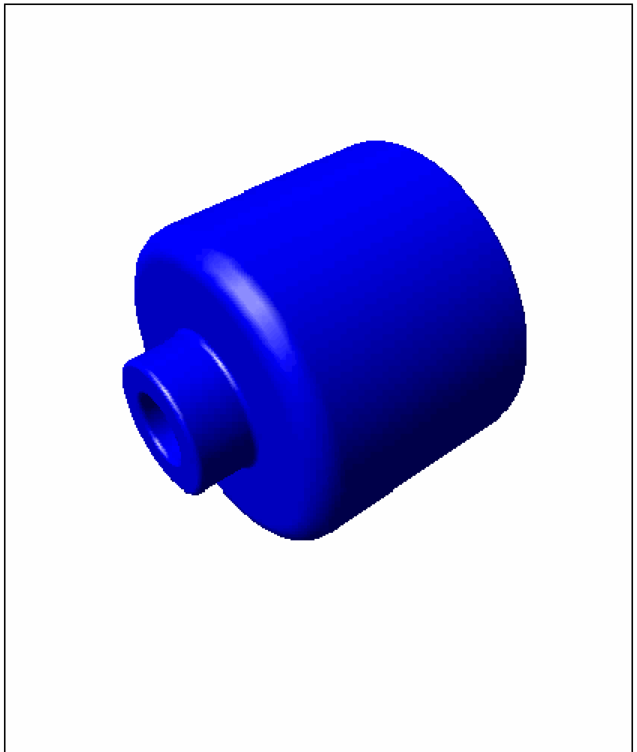
$$\omega'_{si} = \omega_{si} - \omega_a$$

$$-\frac{N_{si}}{N_{ri}} = \frac{\omega_{ri} - \omega_{ai}}{\omega_{si} - \omega_{ai}}$$

## Finding the train ratio: Say the ring is grounded, sun = input, arm = output

$$-\frac{N_{si}}{N_{ri}} = \frac{0 - \omega_{ai}}{\omega_{si} - \omega_{ai}} \longrightarrow \frac{\omega_{ai}}{\omega_{si}} = \frac{N_s}{N_R + N_s}$$

# Planetary gear systems: Arm as output



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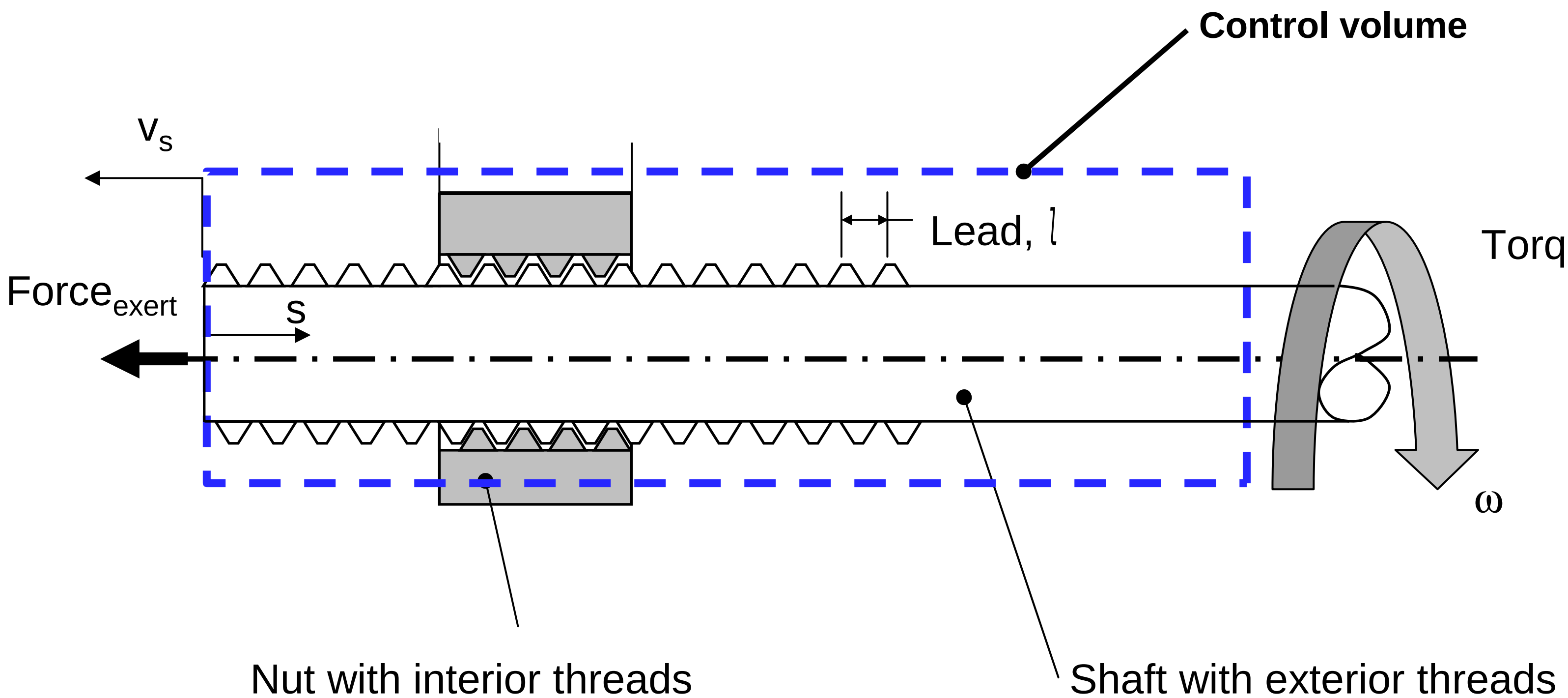
# THREADED MECHANISMS

# Threaded mechanisms: Geometry

## Threaded mechanisms are used in applications such as:

- **Bolts**
- **Lead screws (i.e. mills and lathes)**

## General threaded mechanism geometry




## Usually, either the nut or the screw is grounded

## Figure above shows the nut grounded

# Threaded mechanisms: Modeling power flow

## From power balance for our control volume:

$$\Sigma P_{in} = \left[\Sigma P_{out}\right] + \Sigma\left(\frac{d(E_{stored})}{dt}\right) \quad \rightarrow \quad P_{applied} = \left[P_{exert} + P_{loss}\right] + \frac{d(E_{stretch})}{dt}$$

| | |
|---|---|
| Power in via work by applied Torque: | $\mathrm{P}_{applied} = \vec{\mathrm{T}}_{\mathrm{applied}}(\vec{\omega}) \cdot \vec{\omega}$ |
| Power out via work done by exerted Force: | $\mathrm{P}_{exert} = \vec{\mathrm{F}}_{\mathrm{exert}}(\vec{\mathrm{v}}) \cdot \vec{\mathrm{v}}$ |
| Power loss due to friction Torque: | $\mathrm{P}_{loss} = \vec{\mathrm{T}}_{\mathrm{friction}}(\vec{\omega}) \cdot \vec{\omega}$ |
| Rate of energy storage in stretched "cylinder": | $\mathrm{P}_{stretch} = \vec{\mathrm{F}}_{\mathrm{stretch}}(\vec{\mathrm{v}}_s) \cdot \vec{\mathrm{v}}_s$ |

From geometry: $\mathrm{v} = \left(\frac{\omega}{2\pi}\right) l$ Lead = $l$