Lecture 13

Commodity Modeling

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Morgan Stanley

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Commodity Modeling

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Trader benefits from low prices

First reported 03/11/2009

Dow Jones & Company Inc

Trafigura: May Have Best Earnings Ever In Fiscal 2009

- SINGAPORE -(Dow Jones)- International commodities trading firm **Trafigura** Beheer B.V. is potentially on track to post its best results ever in fiscal 2009 on lower oil prices and contango markets, a company executive said Wednesday.

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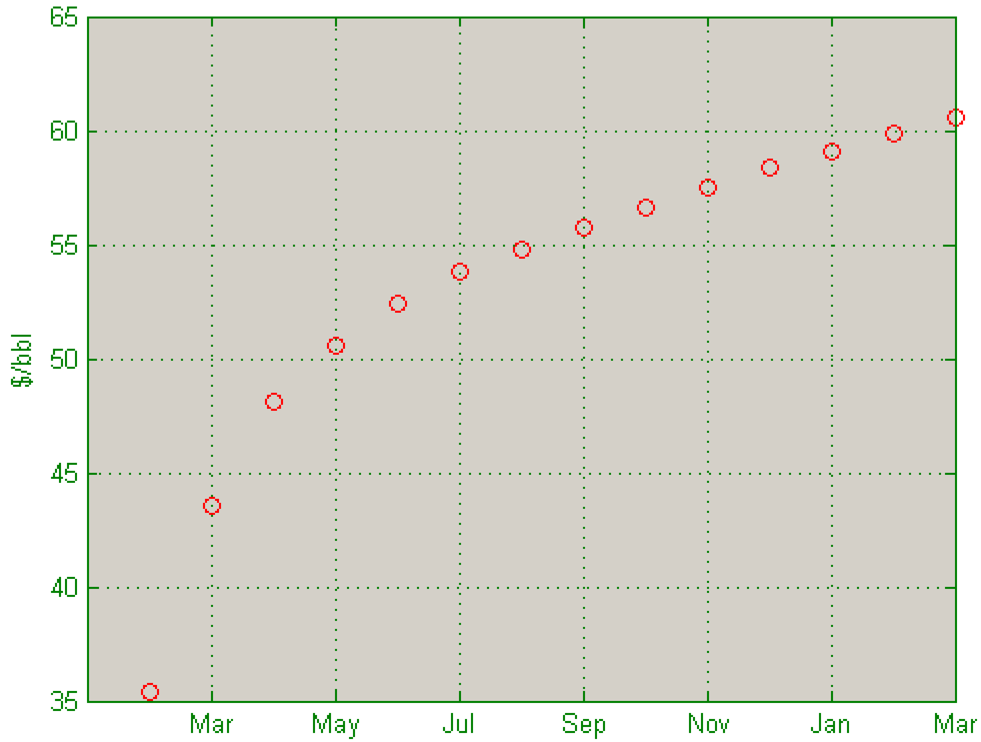
WTI futures contracts: Jan. 15, 2009



Source: Bloomberg

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Trading in Contango Markets

$F_{Feb\,'09} = 35\ \$/bbl$ $\qquad$ $F_{Feb\,'10} = 60\ \$/bbl$

Strategy: On Jan. 15, 2009

- Borrow $35 ➔ Buy 1 bbl ➔ Store
- Short Feb' 10 futures contract (1 bbl)
- Lock-in profit: $25 - Interest Payment
  - *Interest Payment = $35*r*
  - *If r = 10%*

    Interest Payment = $3.5/bbl *Profit = $21.5/bbl*

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Summary: to generate profit

- Needed asset (storage)
- Needed strategy:
  – Long Feb' 09 contract
  – Short Feb' 10 contract
  – Or long Feb-Feb calendar spread

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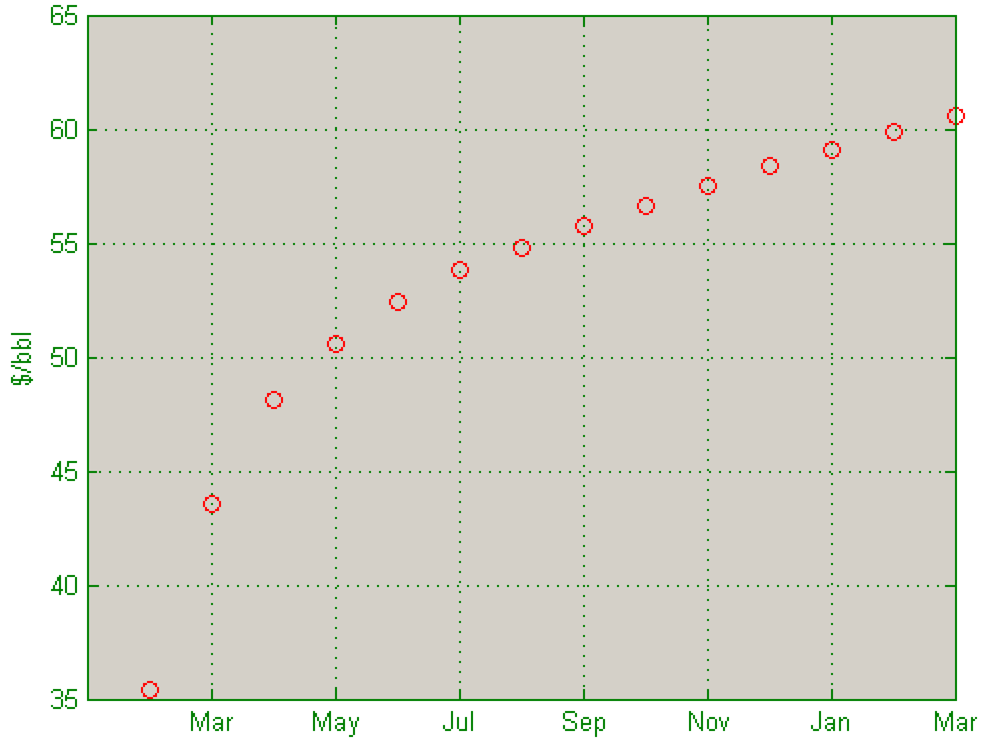
What if you need to lease storage from Aug to Dec How much will you pay for this lease on Jan 1?

- F_{Aug} = 55 $/bbl F_{Dec} = 58 $/bbl



Source: Bloomberg

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This is what the trader will do

On Jan 1

- Buy Aug/Dec spread:
  - Long Aug futures contract
  - Short Dec futures contract
- On Aug 1
  - buy 1 bbl for $55/bbl and store it
- Wait till Dec and then sell 1bbl for $58
- Lock-in $3/bbl. Can pay for storage up to $3/bbl

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This is what the quant will do

- On Jan 1 sell Aug/Dec spread option:

$$Payout\_at\_exercise = \left[F_{Dec} - F_{Aug} \right]^{+}$$

- Exercise date Jul 31
- Interest rates are ignored for simplicity (should not be)

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Why is this better?

- The value of this calendar spread option

$$V = \left(F_{Dec} N\left(d_1\right) - F_{Aug} N\left(d_2\right) \right) \cdot DiscFactor$$

$$d_1 = \left[\log \frac{F_{Dec}}{F_{Aug}} + \frac{\sigma^2}{2} T \right] \Big/ \sigma\sqrt{T} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

– *V = 4.4677 $/bbl*

The value is always greater than the spread because the spread is its intrinsic value

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The benefit:

- Storage bid can be increased to \$4.46/bbl increasing the likelihood of winning the deal. We can also keep a greater profit.
- Is there the risk? What if on Jul 31

  $F_{Aug} = 65$ \$/bbl $\quad\quad F_{Dec} = 80$ \$/bbl

  and we owe \$15/bbl to the option holder

- No worry: We have storage➔ On Jul 31

  Buy Aug crude for 65 \$/bbl and simultaneously

  Sell Dec crude for 80 \$/bbl using Dec futures contract

  Lock-in \$15 \$/bbl to repay option holder

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In reality …

- Sell portfolio of spread option
- Satisfy a number of physical constraints
  - Injection rates
  - Withdarawal rates
  - Do not inject more than max capacity
  - Do not withdraw from the empty tank
  - etc

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Storage optimization

- Find

$$V = \max_{x,v,y,z}\{\sum_{i<j} x_{i,j}S_{i,j} + \sum_{i<j} v_{i,j}U_{i,j} - \sum_{i} y_i F_i + \sum_{j} z_j F_j\}$$

$$x \geq 0, v \geq 0, y \geq 0, z \geq 0$$

- F_i, F_j - today's futures prices for contracts expiring at times T_i and T_j
- y_i, z_j - volumes committed today for injection at time T_i, or withdrawal at T_j

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Storage Optimization

- $S_{i,j}$ - is the value of the option to inject at time and withdraw at time

$$Payout\_at\_exercise = \max\left\{F_j - F_i - Cost,\ 0\right\}$$

- $U_{i,j}$ - is the value of the option to withdraw at time and inject at later time

$$Payout\_at\_exercise = \max\left\{F_i - F_j - Cost,\ 0\right\}$$

- $x_{i,j}, v_{i,j}$- option volumes sold against the storage today

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Constraints

- Let's introduce Boolean "in-the-money at exercise" variables

$$\Omega_{i,j}^{S} = \begin{cases} 1 & \text{if option } S_{i,j} \text{ expires in-the-money} \\ 0 & \text{otherwise} \end{cases}$$

$$\Omega_{i,j}^{U} = \begin{cases} 1 & \text{if option } \mathrm{U}_{i,j} \text{ expires in-the-money} \\ 0 & \text{otherwise} \end{cases}$$

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Constraints

- Injection constraints

$$\sum_{i<j} x_{i,j}\Omega^S_{i,j} - \sum_{\ell<i} x_{\ell,i}\Omega^S_{i,j} + \sum_{\ell<i} v_{\ell,i}\Omega^U_{i,j} - \sum_{i<j} v_{i,j}\Omega^U_{i,j} + y_i - z_i \le I_i \qquad i = 1,\ldots,N$$

- Withdrawal constraints

$$\sum_{i<j} x_{i,j}\Omega^S_{i,j} - \sum_{\ell<i} x_{\ell,i}\Omega^S_{i,j} + \sum_{\ell<i} v_{\ell,i}\Omega^U_{i,j} - \sum_{i<j} v_{i,j}\Omega^U_{i,j} + y_i - z_i \ge -W_i \qquad i = 1,\ldots,N$$

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Constraints

- Maximum capacity constraints

$$C_0 + \sum_{k \le i} \left\{ \sum_{j>i} x_{k,j} \Omega^S_{k,j} - \sum_{j>i} v_{k,j} \Omega^U_{k,j} + y_k - z_k \right\} \le C_i^{\max} \qquad i = 1, \ldots, N$$

- Minimum capacity constraints

$$C_0 + \sum_{k \le i} \left\{ \sum_{j>i} x_{k,j} \Omega^S_{k,j} - \sum_{j>i} v_{k,j} \Omega^U_{k,j} + y_k - z_k \right\} \ge C_i^{\min} \qquad i = 1, \ldots, N$$

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Solution

- Approximation
- Monte-Carlo simulation
- Alternative approach: Stochastic control

Rene Carmona & Michael Ludkovski, 2010. "**Valuation of energy storage: an optimal switching approach**,"Quantitative Finance, Taylor and Francis Journals, vol. 10(4), pages 359-374.

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Additional complications

- There is no spread option market now: we cannot sell spread option directly
- We must design a strategy of replicating selling the spread option
- Similar to Black-Scholes delta-hedging strategy

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Power Plant

- Spark Spread Option
- Merchant Power Plant
  - Should be run if the market price of power is higher than the cost of fuel plus variable operating costs

Net Profit from this operating strategy is:

$$\Pi = \max\left(Price_{Power} - \frac{Heat\_Rate}{1000} Price_{Fuel} - Variable\_Costs, 0 \right)$$

Operating a merchant power plant is financially equivalent to owning a portfolio of daily options on spreads between electricity and fuel (spark spread options)

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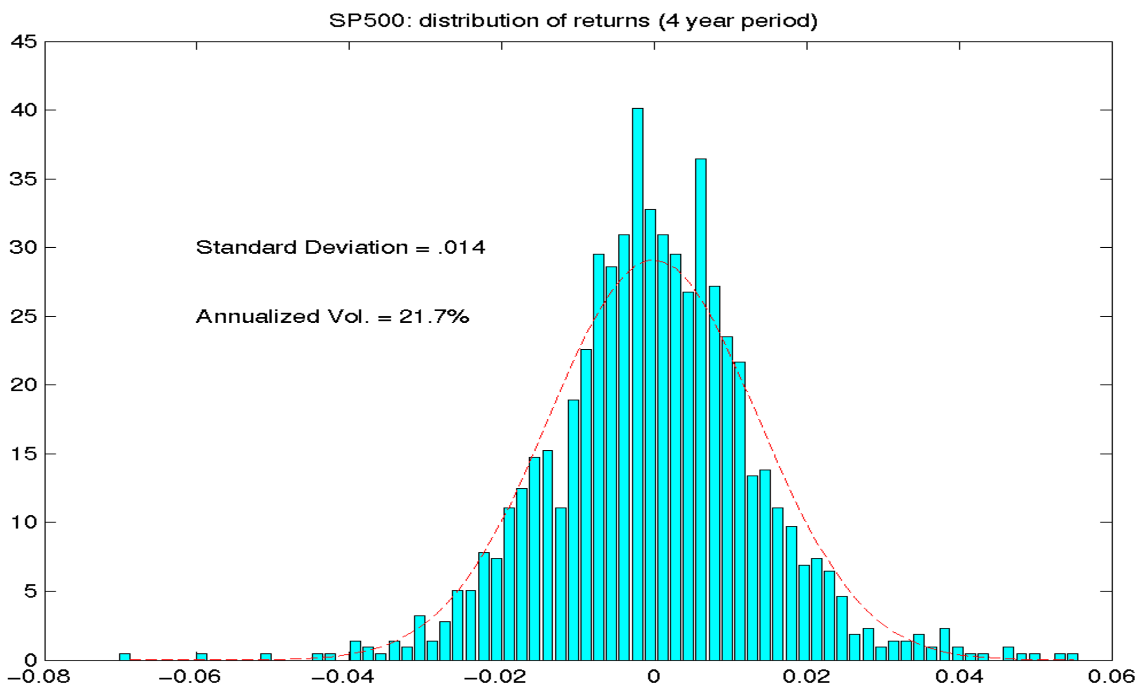
Properties of energy prices

- Behavior of energy prices is unique

Example 1: Fat Tails of distributions



Source: Eydeland, Wolyniec

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Properties of energy prices

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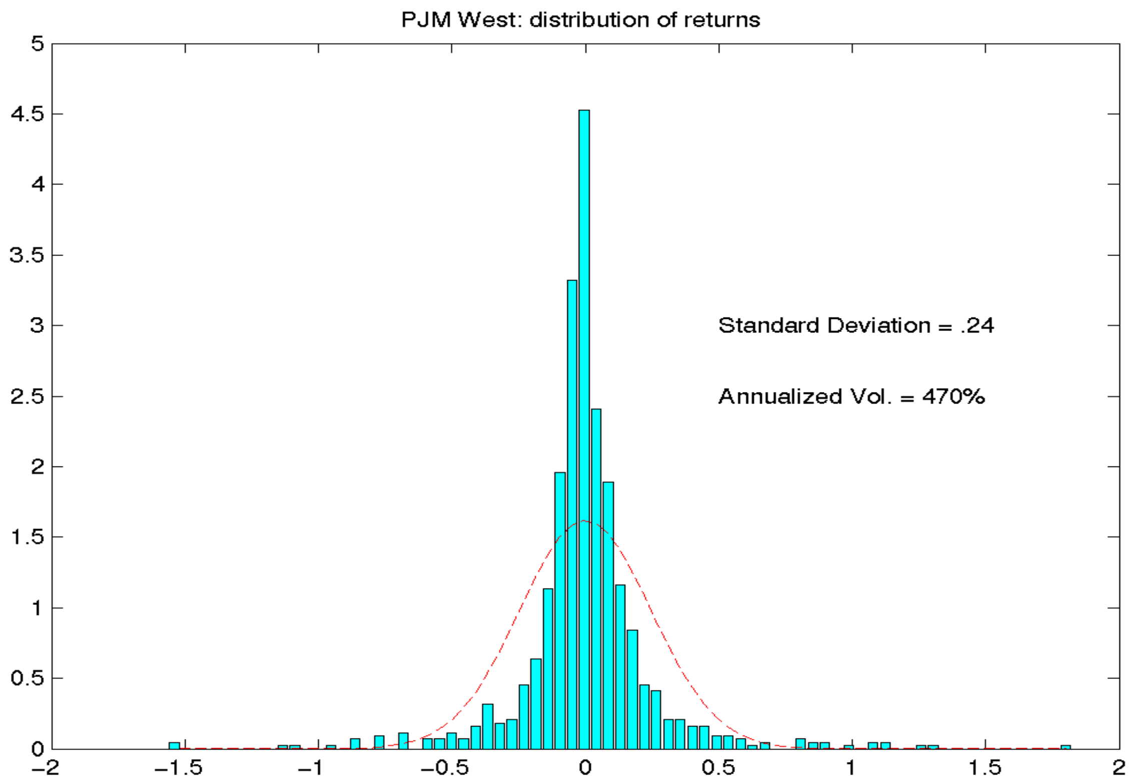

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Properties of energy prices

Distribution Parameters (A. Werner, Risk Management in the Electricity Market, 2003)

	Annual. Volatility	Skewness	Kurtosis
Nord Pool	182%	1.468	26.34
NP 6.p.m.	238%	2.079	76.82
DAX	23%	0.004	3.33

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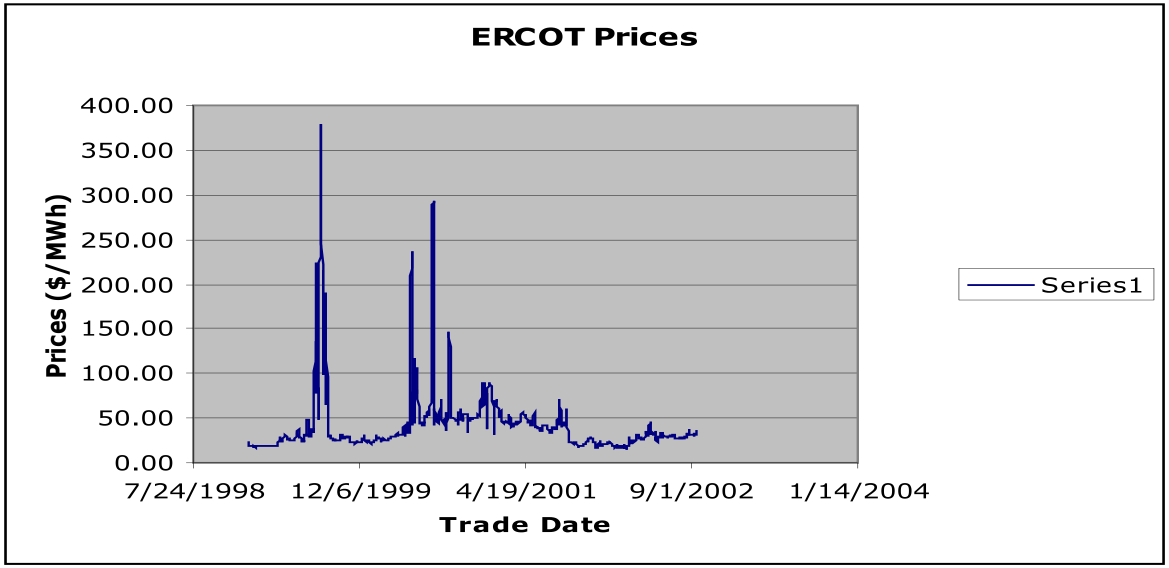
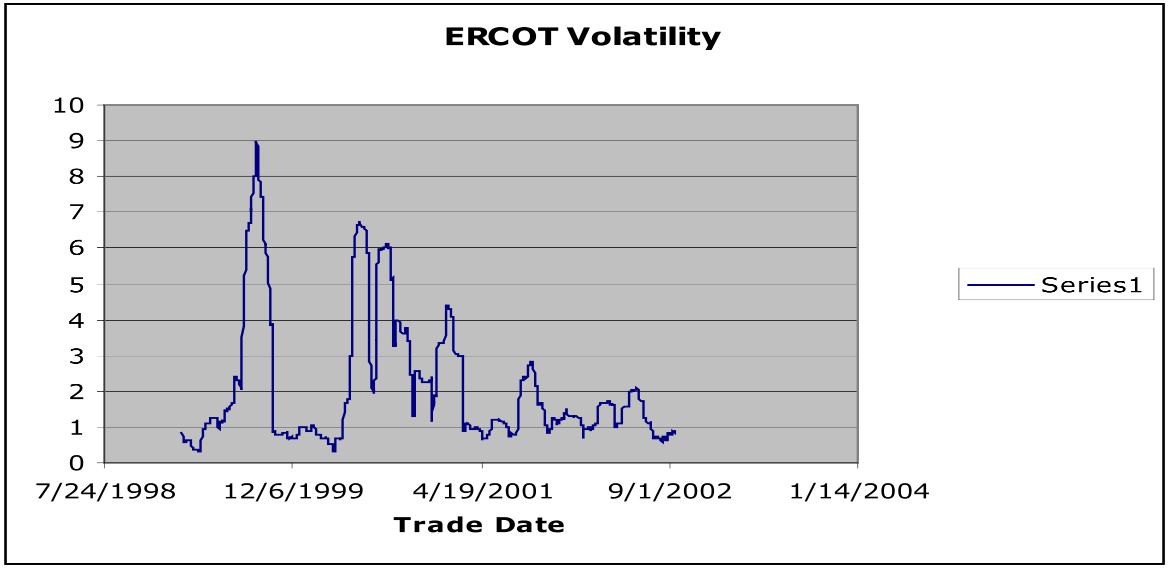
Special properties of electricity prices: spikes, high volatility

Source: Eydeland, Wolyniec





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Special properties of electricity prices

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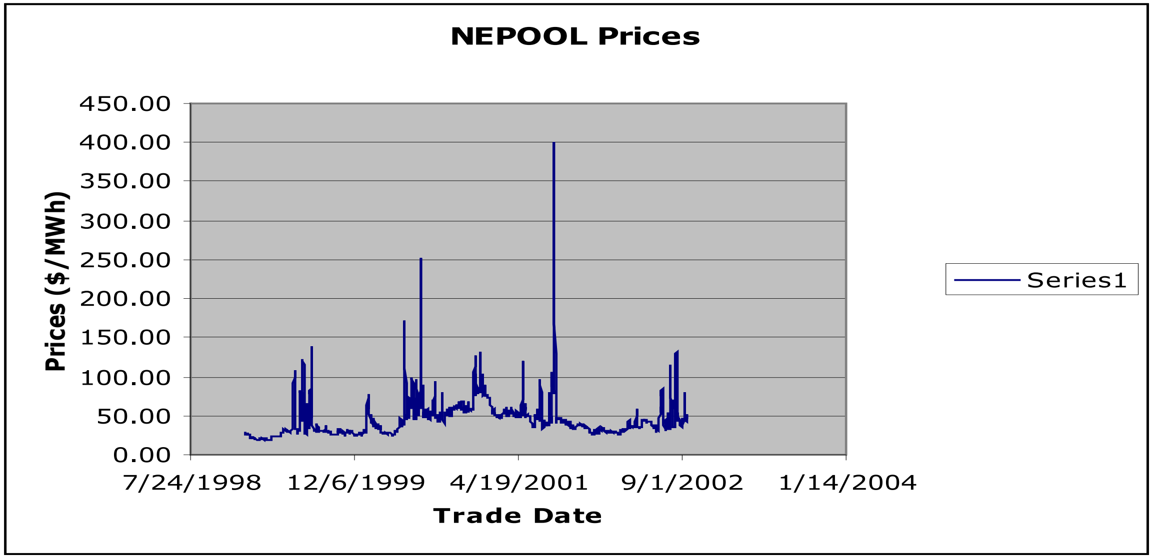

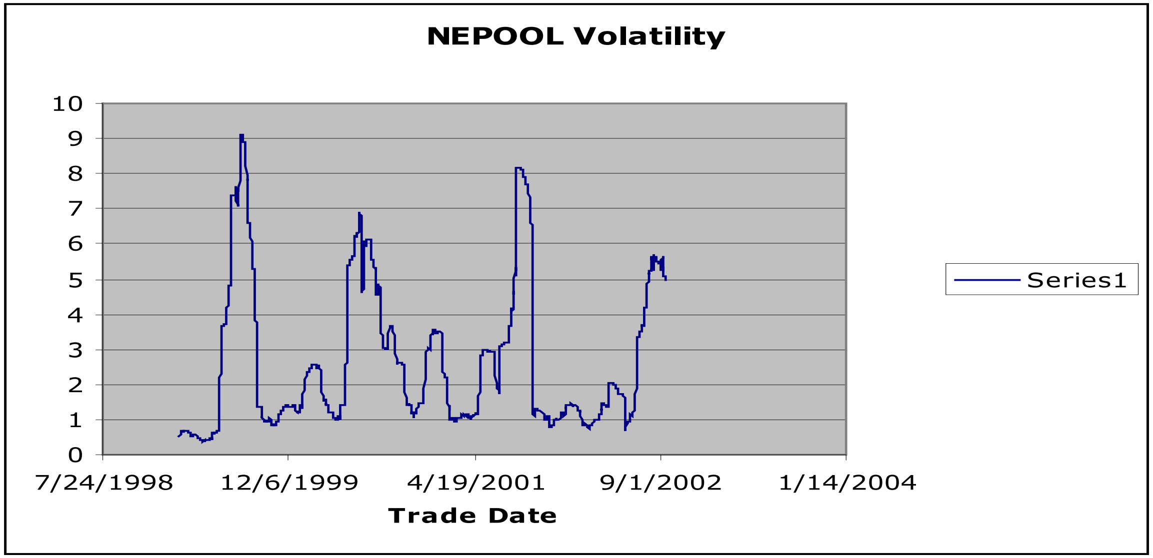

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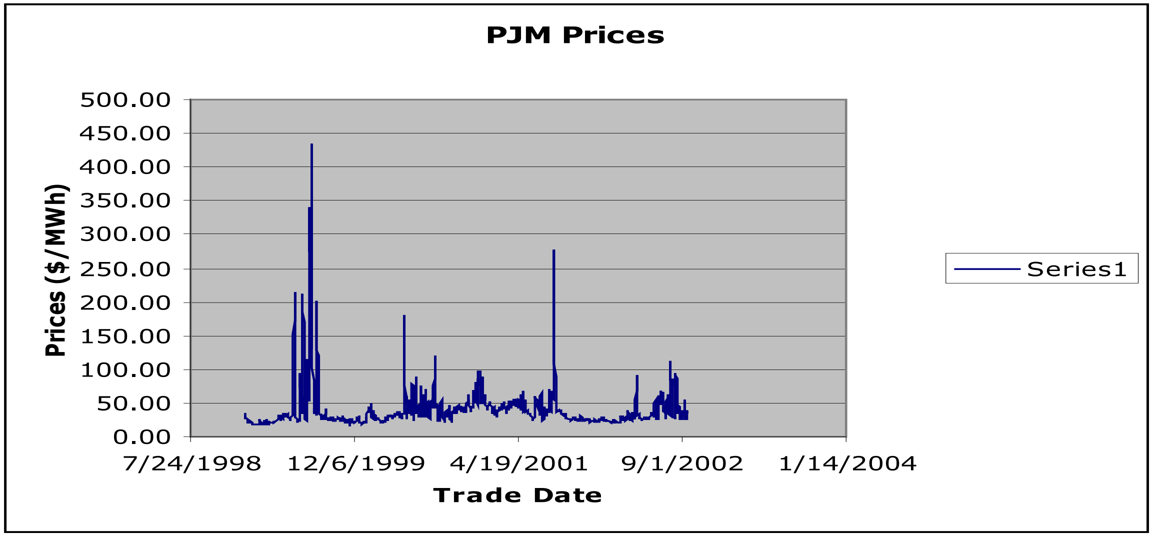

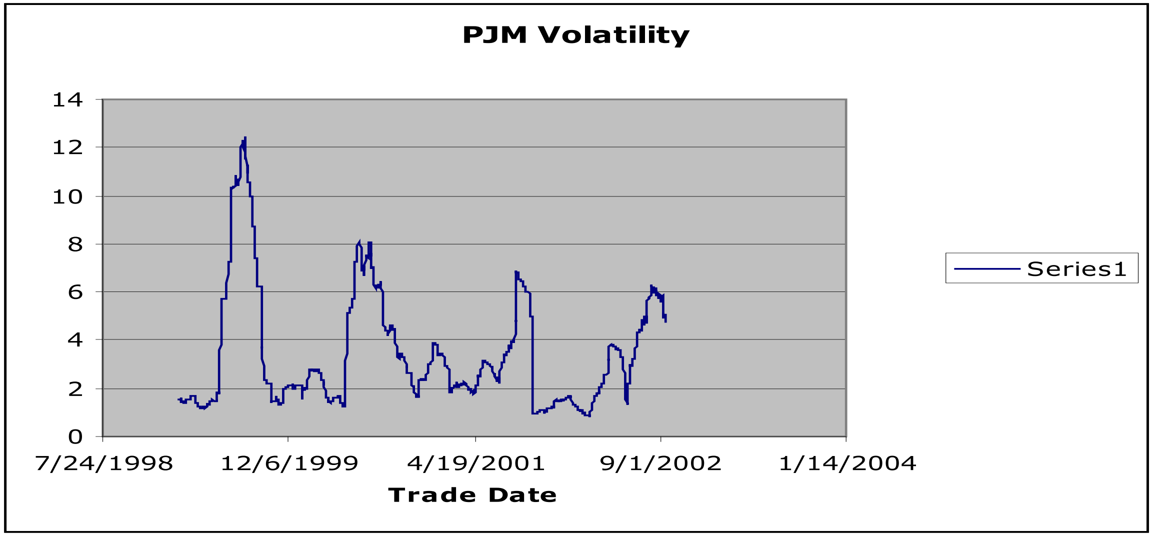

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Behavior of power prices

- Mean reversion
- spikes
- high kurtosis
- regime switching
- lack of data
- non-stationarity

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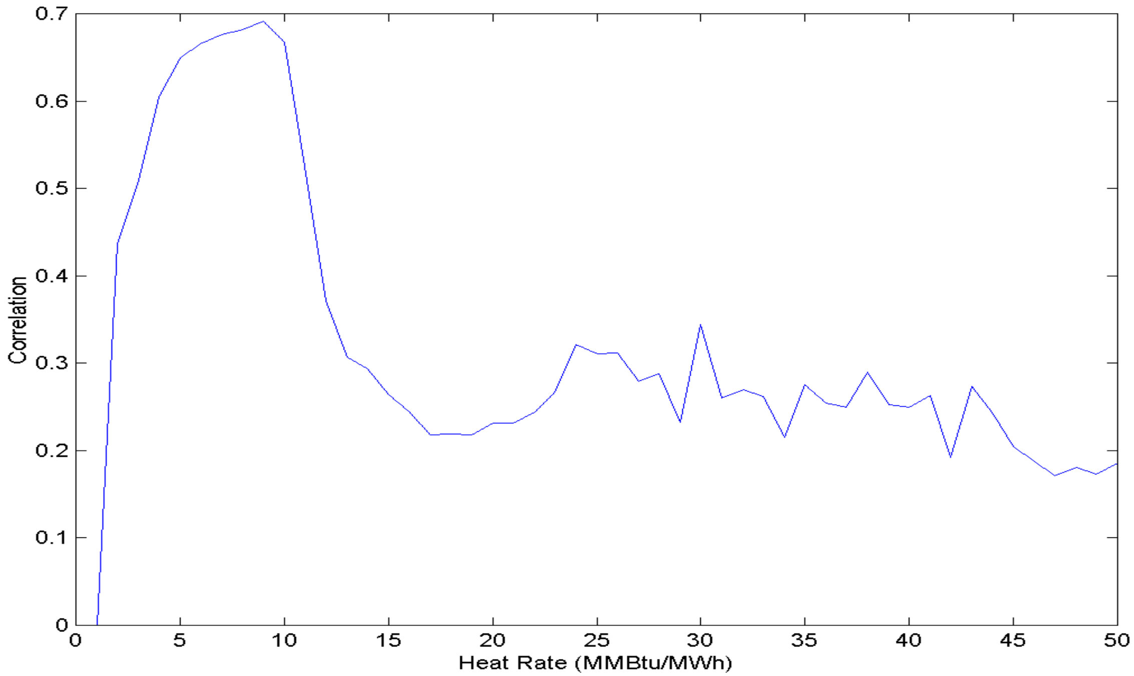
Joint distribution: power/NG correlation structure

Correlation between power and gas also has unique structure. If the model does not capture this structure, it may misprice spread options (tolling contracts, power plants, etc.)



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Models

- Spot Processes
- GBM

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

- GBM with mean reversion

$$\frac{dS_t}{S_t} = \kappa(\theta - \log S_t)dt + \sigma\, dW_t$$

- + jumps

$$\frac{dS_t}{S_t} = (\mu - \lambda k)dt + \sigma\, dW_t + (Y_t - 1)dq_t$$

- + jumps and mean reversion

$$\frac{dS_t}{S_t} = (\mu - \lambda k - \log S_t)dt + \sigma\, dW_t + (Y_t - 1)dq_t$$

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More complicated models

- Models with stochastic convenience yield
- Models with stochastic volatility
- Regime switching models
- Models with multiple jump processes
- Various term structure models

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Spot Precesses: Cons

Difficult to use for power products due to non-storability:

- No no-arbitrage argument
- How to price forward contracts and options?
  - In the case of storable commodities (NG, CL) we need convenience yield.
  - Calibration is difficult to implement due to overlapping data.
  - Cannot model the correlation structure between forward contracts.
  - Cannot model complex volatility structures.
  - Spot processes without jumps or stochastic volatility generate unrealistic power price distributions.
  - Cannot capture complex power/gas correlation structure.

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A different approach

Hybrid Model: Stack Method

Price formation mechanism: Bid stack

Generator 1.

Price ($/MWh)	20	25	30	35	50
Volume (MWh)		50	100	200	400	600

Generator 2.

Price ($/MWh)	18	40	100
Volume (MWh)		100	200	500

Source: Eydeland, Wolyniec

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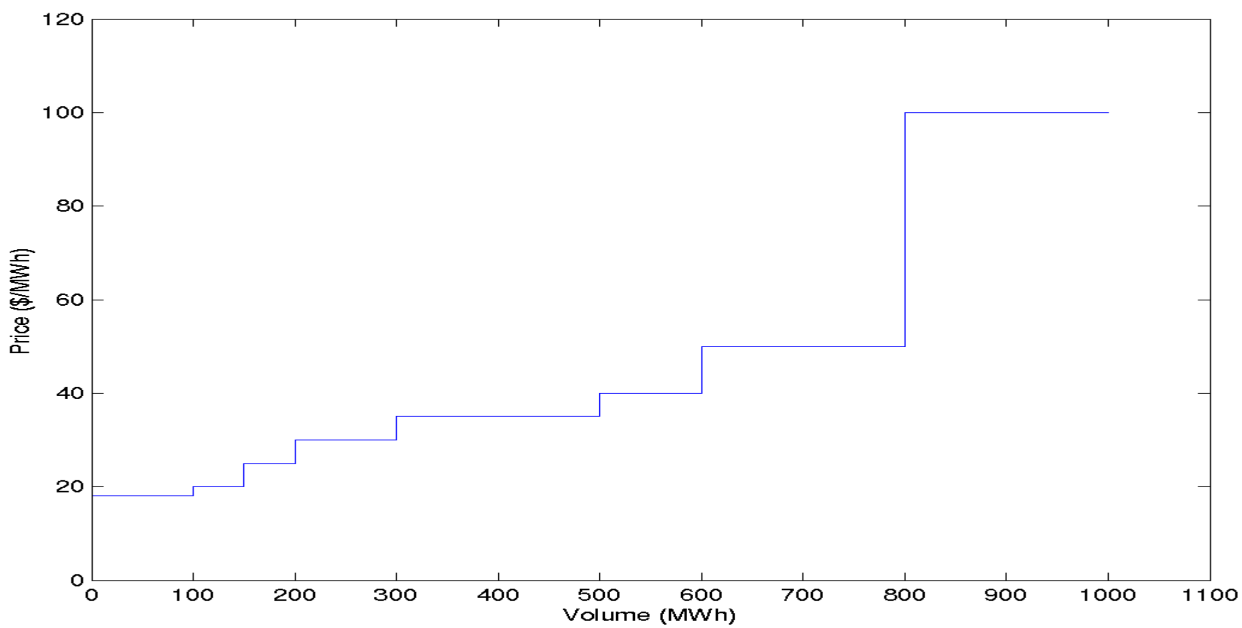
Hybrid Models: Stack Method

Bid stack:

Price ($/MWh)	18	20	25	30	35	40	50	100
Volume (MWh)	100	150	200	300	500	600	800	1100

Source: Eydeland, Wolyniec



$$P_t = S^{bid}(D_t)$$

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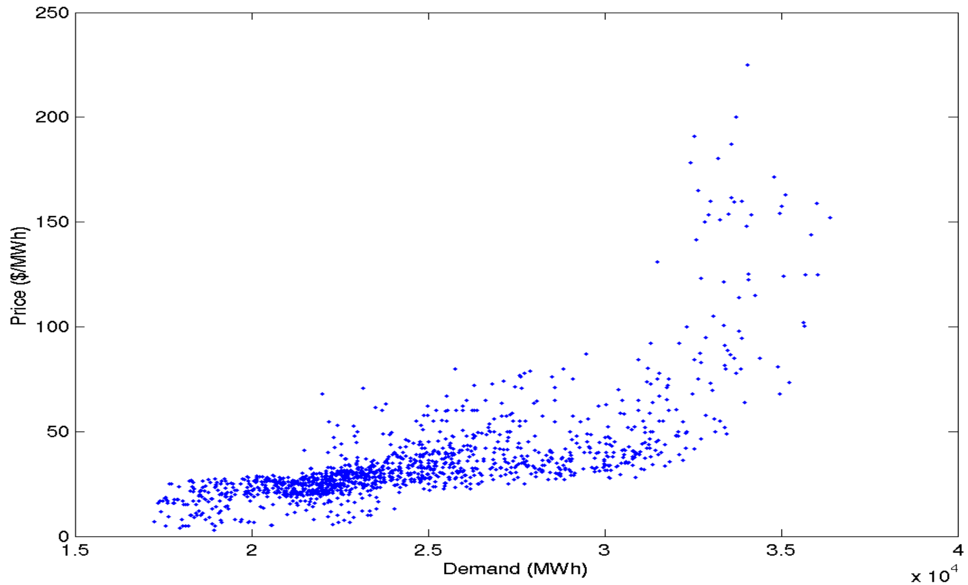
Drivers:

1. Demand



Source: Eydeland, Wolyniec

2. Fuel Prices
3. Outages

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How to build the bid stack?

1. Fuel + Outages $\longrightarrow$ Generation Stack

2. Generation stack $\longrightarrow$ Bid Stack

Transformation at step 2 matches market data and preserves higher moments of price distribution (skewness, kurtosis)

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Fuel Model

Group 1

- Natural Gas
- #2 Heating Oil
- #6 Fuel Oil (with different sulfur concentration)
- Coal
- Jet Fuel
- Diesel
- Methane
- Liquefied Natural Gas (LNG)
- Etc.

Group 2

- Nuclear
- Hydro
- Solar
- Wind
- Biomass
- Etc.

Prices of Group 1 fuels are modeled using term structure models, matching forward prices, option prices and correlation structure

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Outage Model

Standard process (e.g., Poisson) utilizing EFOR (Equivalent Forced Outage Rate)

As a result for each time T we have an outage vector

$$\Omega_T = (\omega_{T,1}, \ldots, \omega_{T,L})$$

$\omega_{T,i}=\mathbf{1}$ if at time $\boldsymbol{T}$ the unit $\boldsymbol{I}$ is experiencing forced outages

$\omega_{T,i}=\mathbf{0}$ otherwise

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Demand

Demand can be modeled as a function of temperature

$$D_t = d(t, \Im_t)$$

Temperature evolution process:

i. evolution of the principal modes
ii. evolution of the daily perturbations

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Power Prices

$$P_T = s_T^{bid}(D_T) \equiv \alpha_1 s^{gen}\left(D_T;\, T, U_T, \Omega_T\left(\alpha_2\lambda\right), E_T, VOM_T, \alpha_3 C_T\right)$$

The constants α chosen to match market data

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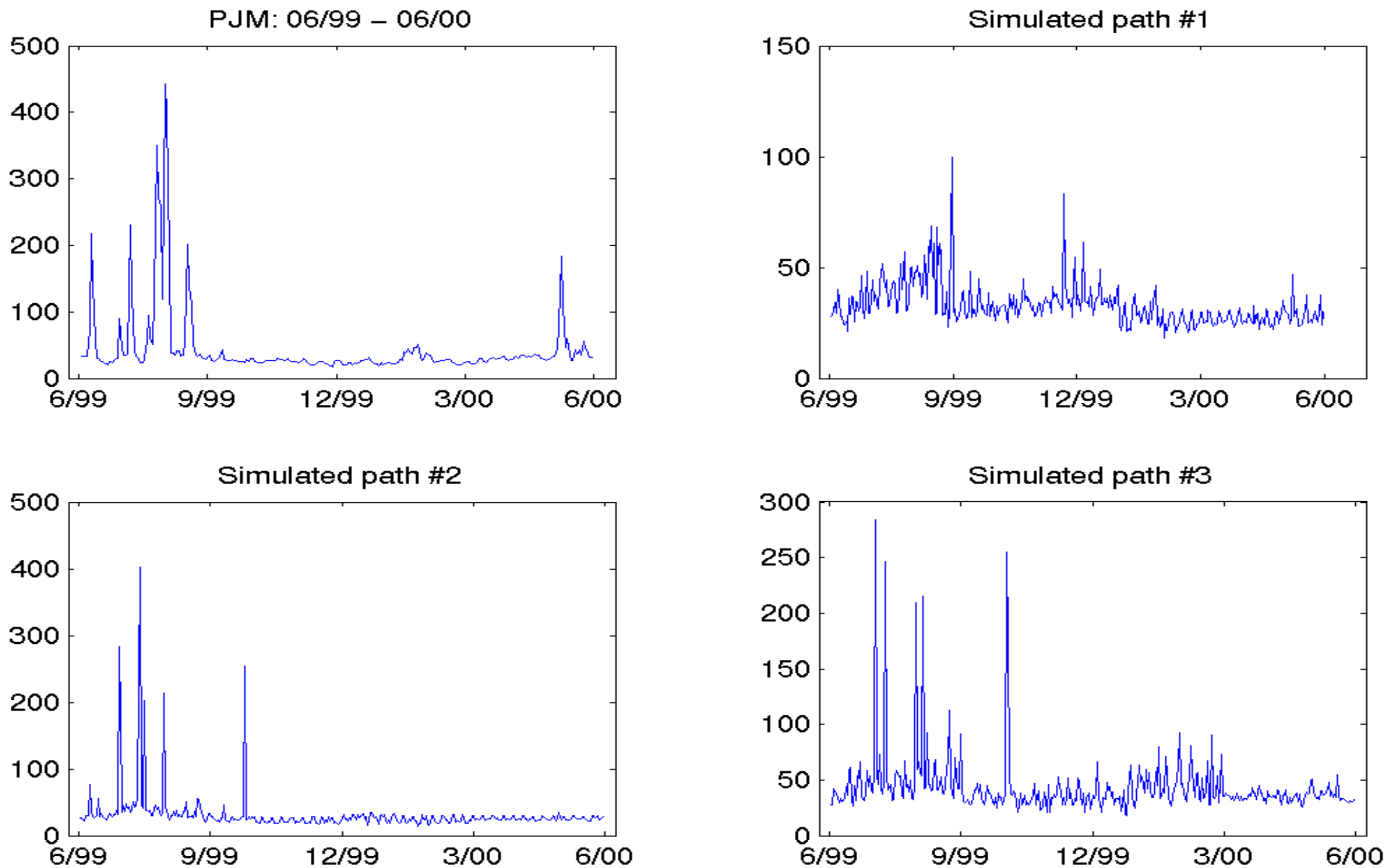
Justification

Source: Eydeland, Wolyniec



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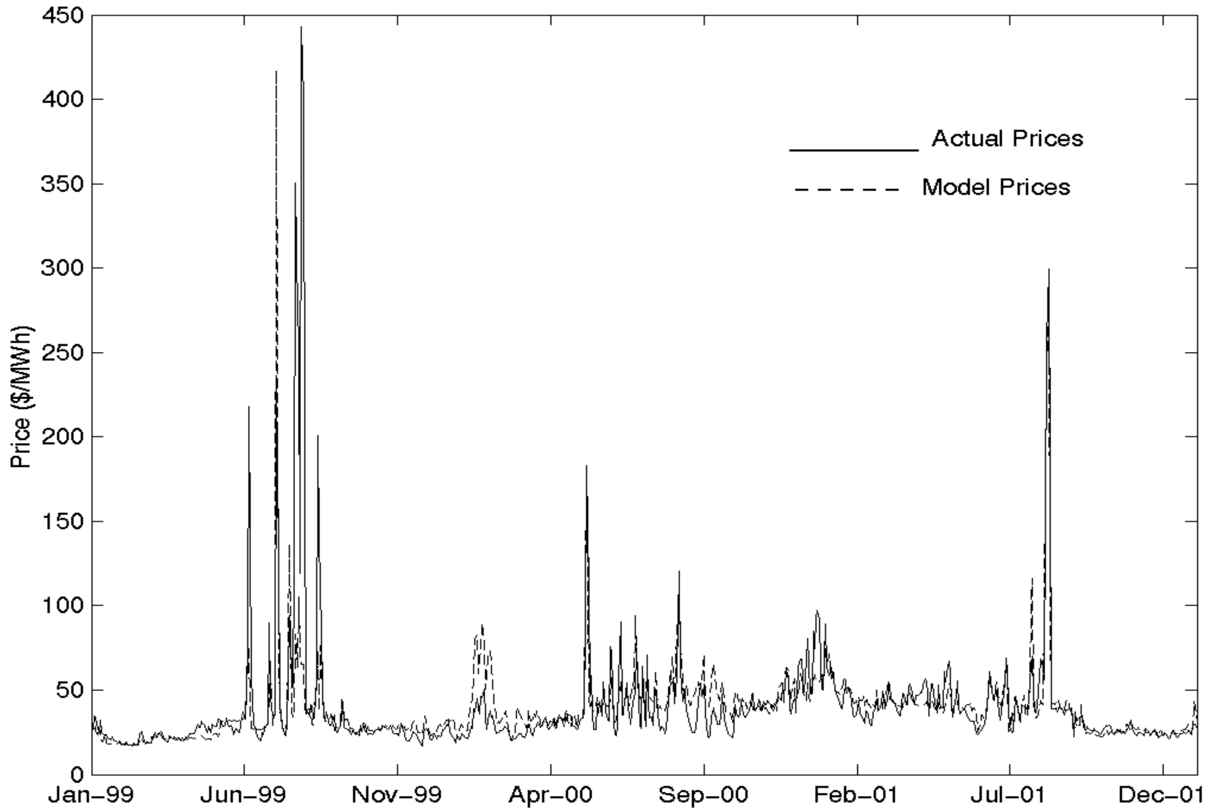
Justification: PJM Prices - actual vs. model

Source: Eydeland, Wolyniec



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Justification

Skewness and kurtosis of PJM price distribution: model vs. empirical data

	Skewness		Kurtosis	
	Model data	Empirical data	Model data	Empirical data
Summer 2000	3.58	3.17	4.77	4.89
Summer 2001	18.13	14.65	25.83	26.46
Winter 2000	.68	1.32	2.02	1.19
Winter 2001	.18	1.54	5.48	1.98

Source: Eydeland, Wolyniec

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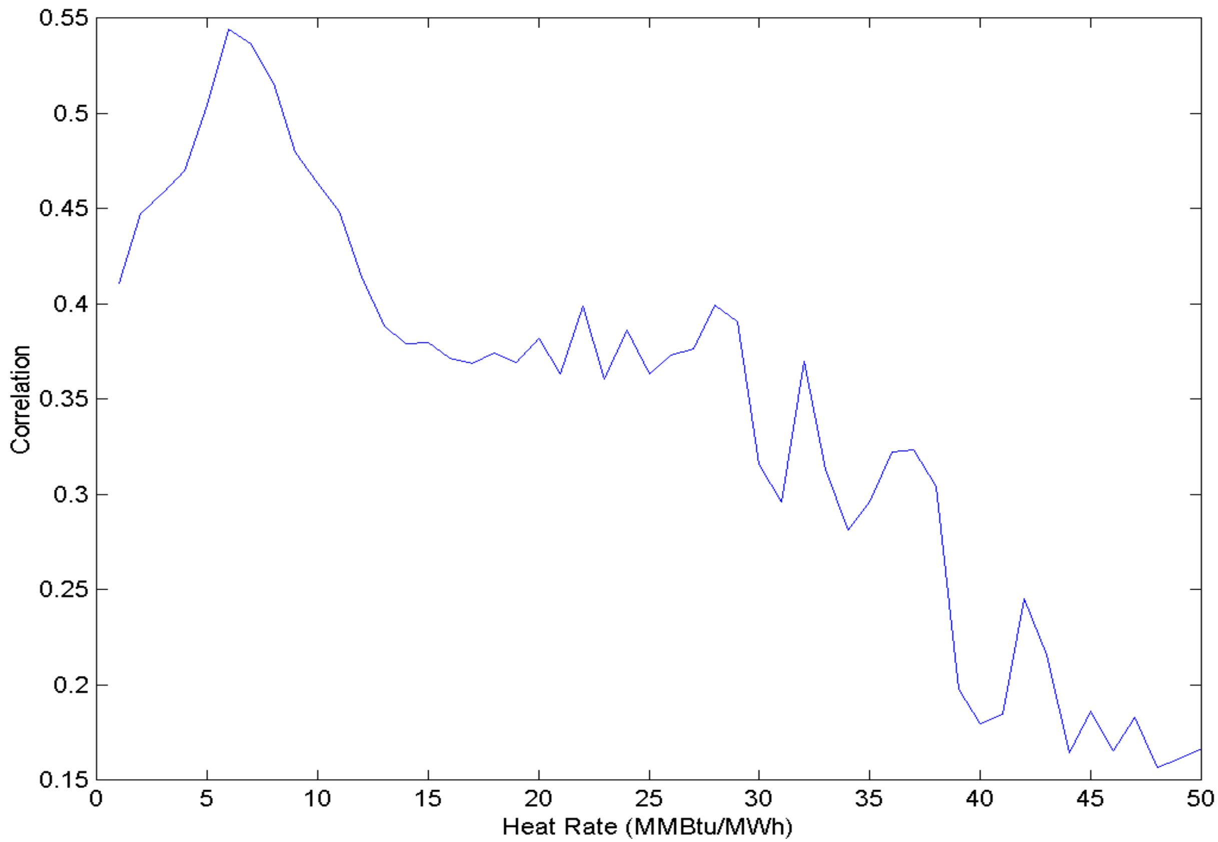
Simulated correlation structure

Source: Eydeland, Wolyniec



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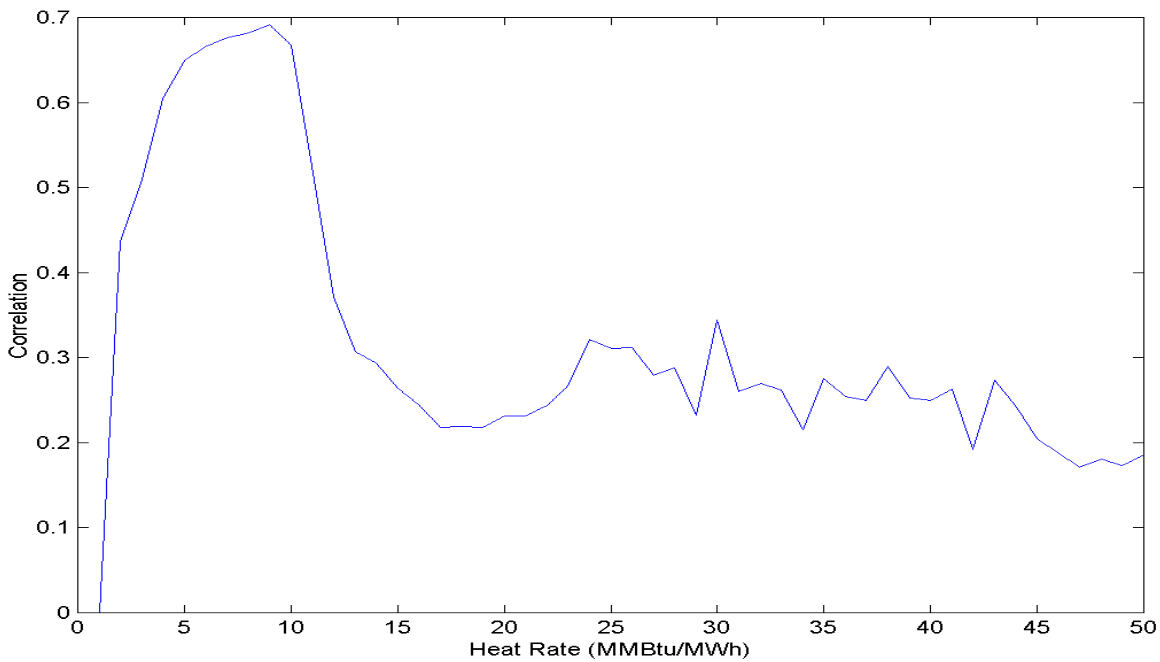
vs actual correlation structure

Source: Eydeland, Wolyniec



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18.S096 Topics in Mathematics with Applications in Finance
Fall 2013

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