Case Study 5: Multivariate Time Series

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Contents

1 VAR Models of Macro Economic Time Series

1.1 Macroeconomic Forecasting Models

In the 1980s, Robert Litterman and Christopher Sims developed important macroeconomic forecasting models based on vector autoregressions(VAR). The models use aggregate macroeconomic variables including:

- Treasury bill rate
- M1 (money supply)
- GNP deflator (inflation)
- real GNP (Gross National Product, economic output)
- real business fixed investment
- unemployment
- trade-weighted value of the dollar
- S&P-500 index (equity market valuation)
- Commodity price index.

With such models, policy makers have the potential to anticipate changes in macroeconomic conditions. Also, incorporating variables reflecting policy actions (e.g., Federal Funds Rate) helps to evaluate the potential impact of policy actions.

There is an extensive literature on VAR modeling; see the citations in Pfaff(2008). The papers of Litterman and Sims in the references provide a good introduction to the mathematiacl framework for specifying vector autoregression models in a Bayesian framework. Sims, extending the model of Litterman, accommodates time-varying variances of the disturbance/innovation terms, and non-Gaussianity of these disturbances.

The analysis in the following sections uses the R package to collect macroeconomic time series and fit vector-autoregressive models to a reduced set of these macroeconomic variables.

1.2 Collecting the Macroeconomic Data

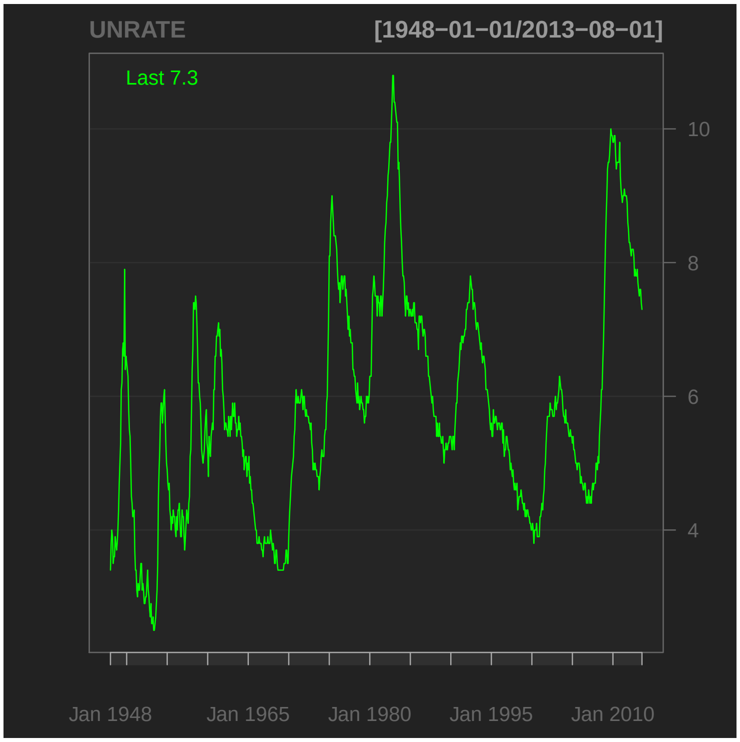
```
> # 1.  Load R Libraries
>
> source("fm_casestudy_0_InstallOrLoadLibraries.r")
> # Collect macro economic data from FRED database
> # Macro Variables
> #
> # UNRATE    unemployment
> # FEDFUNDS  Federal Funds Rate
> # TB3MS     Treasury Bill Rate
> # CPIAUCSL  CPI Index All Urban Customers All Items
>
> # M1SL      M1
> # GDPDEF    GNP deflator
> # GDP       real GNP
> # GPDI      real business fixed investment
>
> # TWEXBMTH  Trade weighted value of dollar
> # SP500     S&P 500 Index
>
> getSymbols("UNRATE", src="FRED")
```

```
[1] "UNRATE"
```

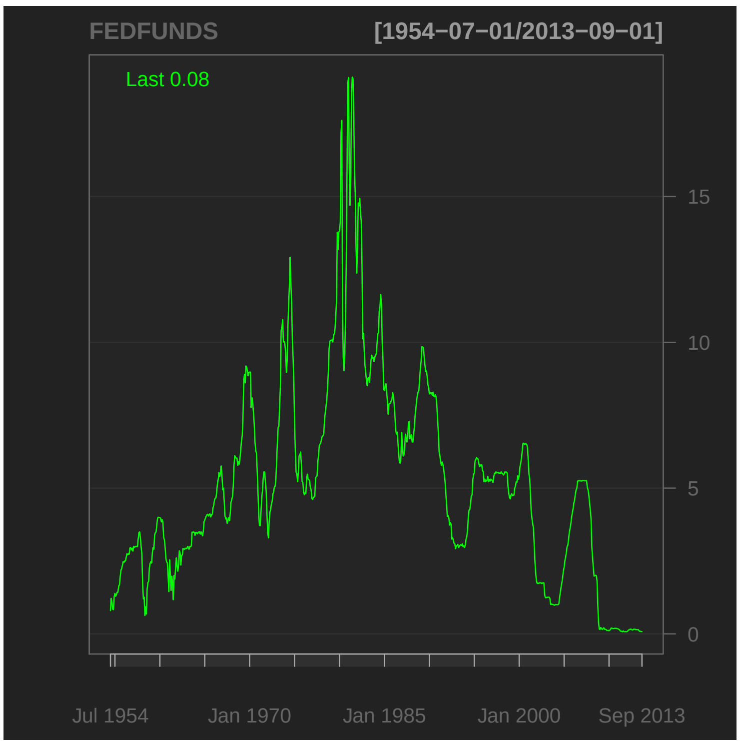
```
> head(UNRATE)
```

```
           UNRATE
1948-01-01    3.4
1948-02-01    3.8
1948-03-01    4.0
1948-04-01    3.9
1948-05-01    3.5
1948-06-01    3.6
```

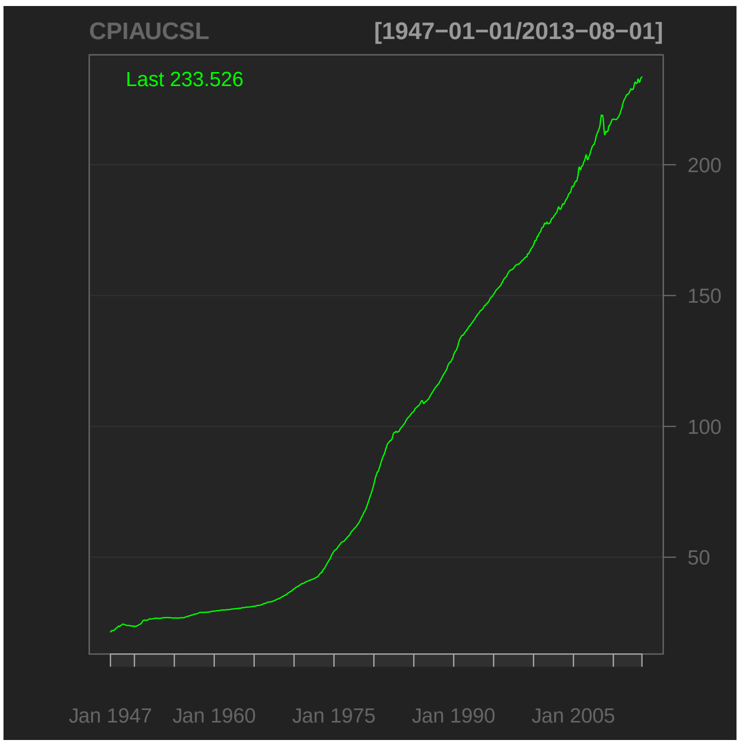
```
> chartSeries(UNRATE)
```


UNRATE
[1948−01−01/2013−08−01]
Last 7.3
10
8
6
4
Jan 1948
Jan 1965
Jan 1980
Jan 1995
Jan 2010

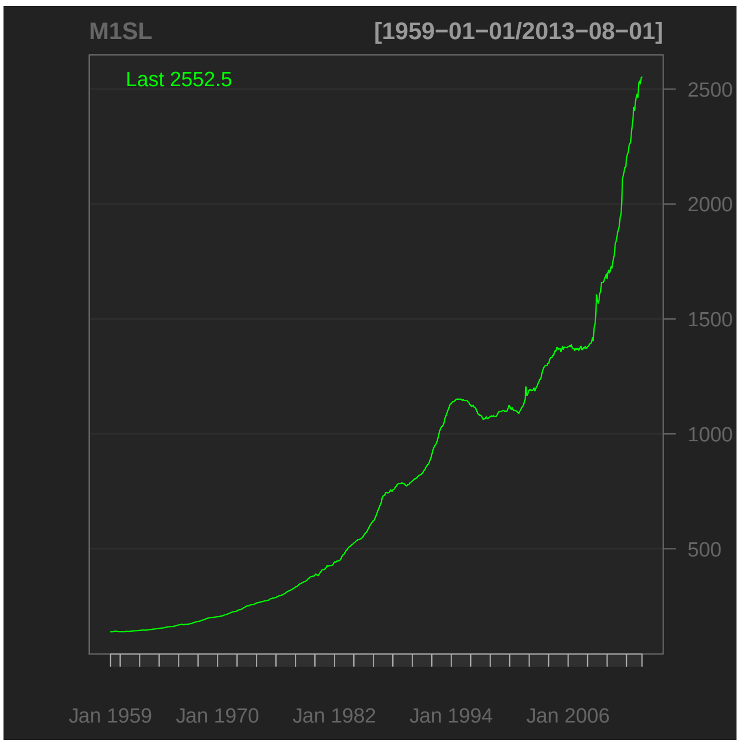
```
> getSymbols("FEDFUNDS", src="FRED")

[1] "FEDFUNDS"

> head(FEDFUNDS)

           FEDFUNDS
1954-07-01     0.80
1954-08-01     1.22
1954-09-01     1.06
1954-10-01     0.85
1954-11-01     0.83
1954-12-01     1.28

> chartSeries(FEDFUNDS)
```



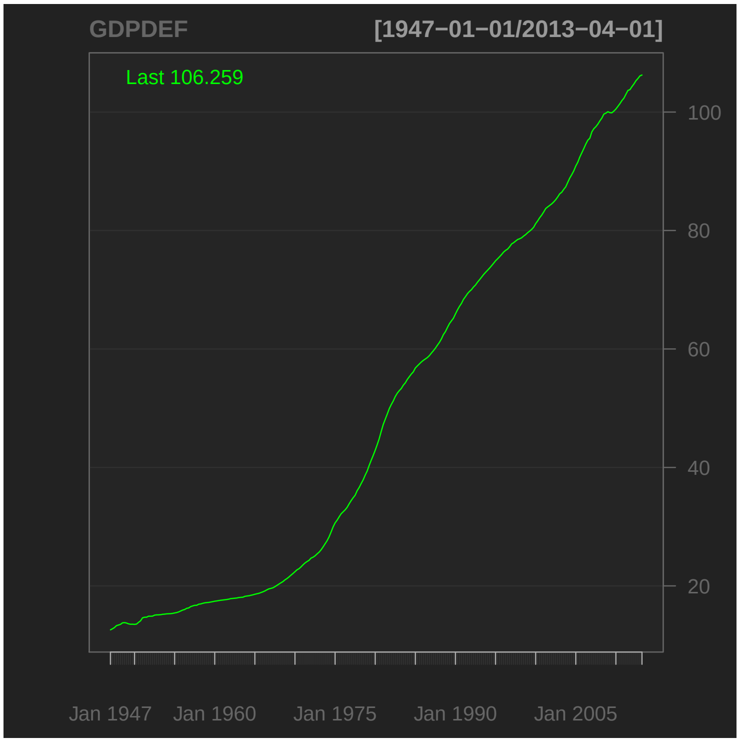
```
> getSymbols("CPIAUCSL", src="FRED")

[1] "CPIAUCSL"

> head(CPIAUCSL)

           CPIAUCSL
1947-01-01    21.48
1947-02-01    21.62
1947-03-01    22.00
1947-04-01    22.00
1947-05-01    21.95
1947-06-01    22.08

> chartSeries(CPIAUCSL)
```



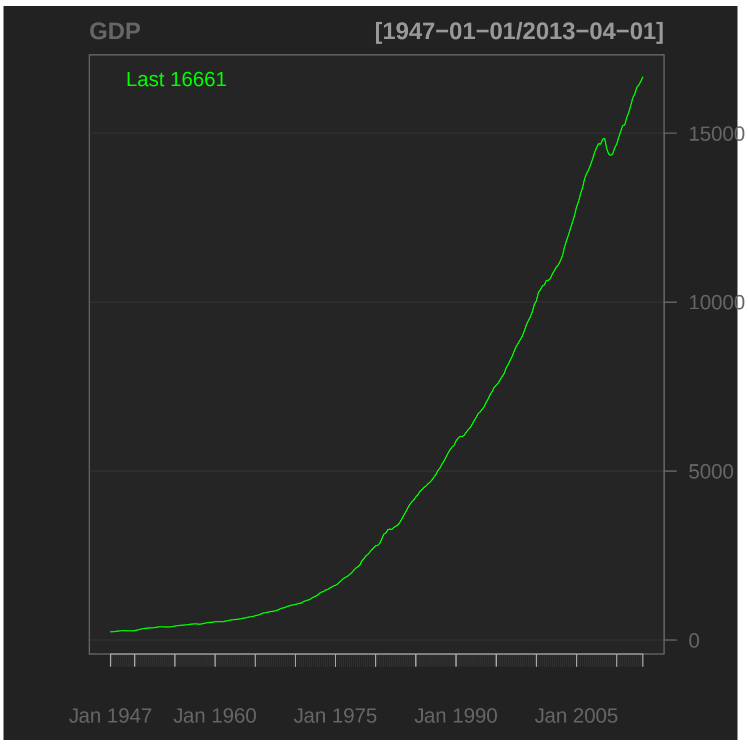
```
> getSymbols("M1SL", src="FRED")

[1] "M1SL"

> head(M1SL)

             M1SL
1959-01-01 138.9
1959-02-01 139.4
1959-03-01 139.7
1959-04-01 139.7
1959-05-01 140.7
1959-06-01 141.2

> chartSeries(M1SL)
```



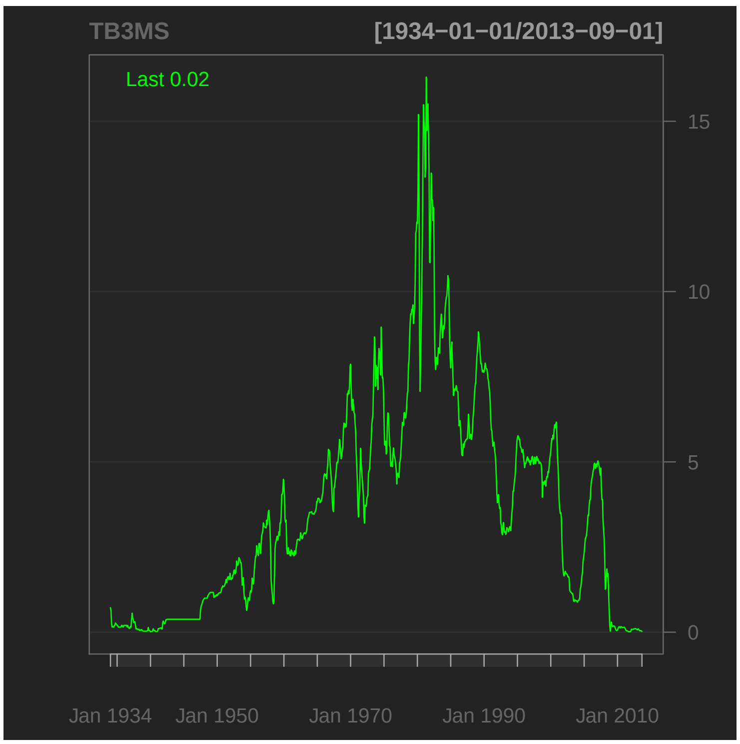
```
> getSymbols("GDPDEF", src="FRED")

[1] "GDPDEF"

> head(GDPDEF)

           GDPDEF
1947-01-01 12.578
1947-04-01 12.757
1947-07-01 12.970
1947-10-01 13.289
1948-01-01 13.392
1948-04-01 13.510

> chartSeries(GDPDEF)
```



```
> getSymbols("GDP", src="FRED")

[1] "GDP"

> head(GDP)

                GDP
1947-01-01 243.1
1947-04-01 246.3
1947-07-01 250.1
1947-10-01 260.3
1948-01-01 266.2
1948-04-01 272.9

> chartSeries(GDP)
```

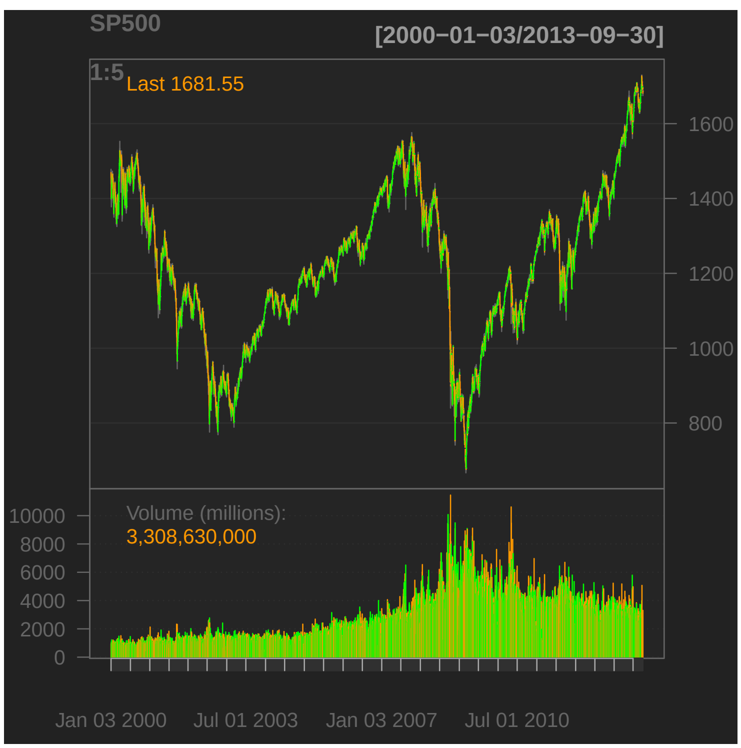


```
> getSymbols("TB3MS", src="FRED")

[1] "TB3MS"

> head(TB3MS)

           TB3MS
1934-01-01  0.72
1934-02-01  0.62
1934-03-01  0.24
1934-04-01  0.15
1934-05-01  0.16
1934-06-01  0.15

> chartSeries(TB3MS)
```



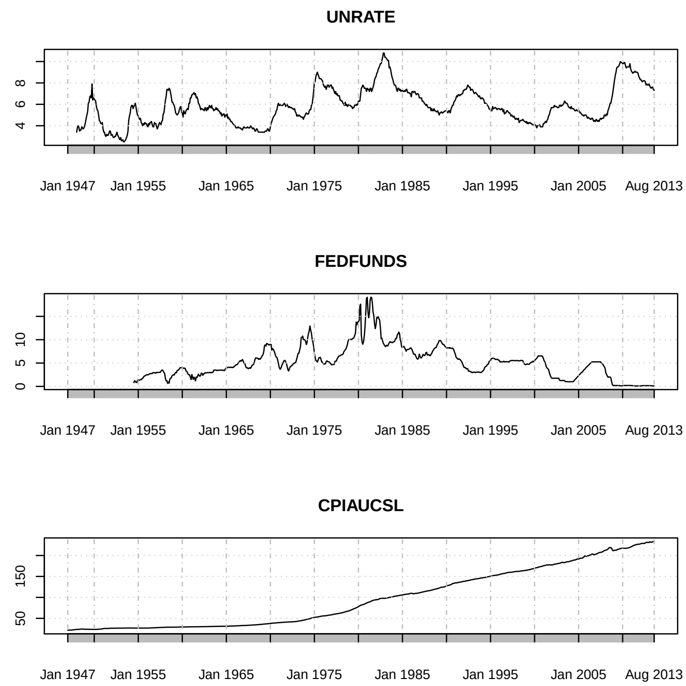
```
> getSymbols("TWEXBMTH", src="FRED")

[1] "TWEXBMTH"

> head(TWEXBMTH)

           TWEXBMTH
1973-01-01  33.9689
1973-02-01  32.5799
1973-03-01  31.5849
1973-04-01  31.7681
1973-05-01  31.5727
1973-06-01  31.0864

> chartSeries(TWEXBMTH)
```

```
> # Collect index data from Yahoo
> #     1.1.1 Set start and end date for collection in YYYYMMDD (numeric) format
> date.start<-20000101
> date.end<-20130930
> #     1.1.2 Collect historical data for S&P 500 Index
> SP500 <- getYahooData("^GSPC", start=date.start, end=date.end)
> head(SP500)

              Open    High     Low   Close     Volume
2000-01-03 1469.25 1478.00 1438.36 1455.22  931800000
2000-01-04 1455.22 1455.22 1397.43 1399.42 1009000000
2000-01-05 1399.42 1413.27 1377.68 1402.11 1085500000
2000-01-06 1402.11 1411.90 1392.10 1403.45 1092300000
2000-01-07 1403.45 1441.47 1400.73 1441.47 1225200000
2000-01-10 1441.47 1464.36 1441.47 1457.60 1064800000

> chartSeries(SP500[,1:5])
```

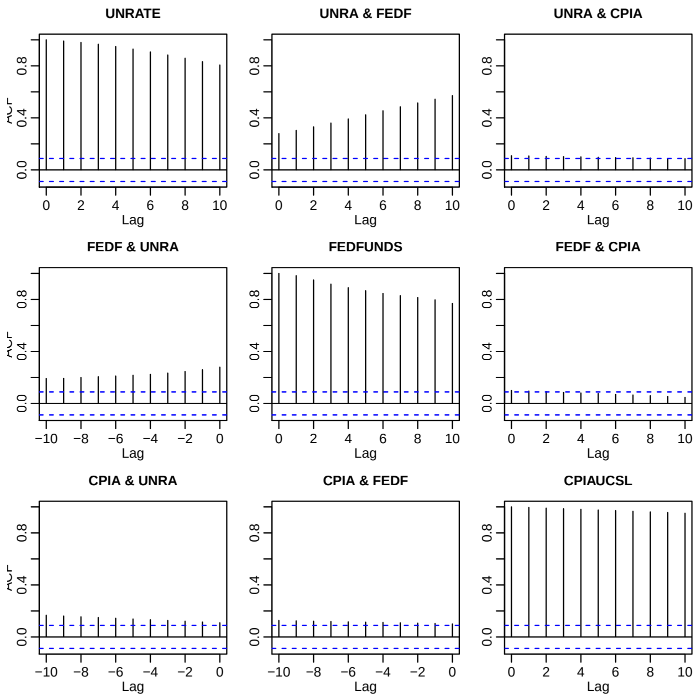


1.3 Ordinary and Partial Autocorrelations of Reduced Set

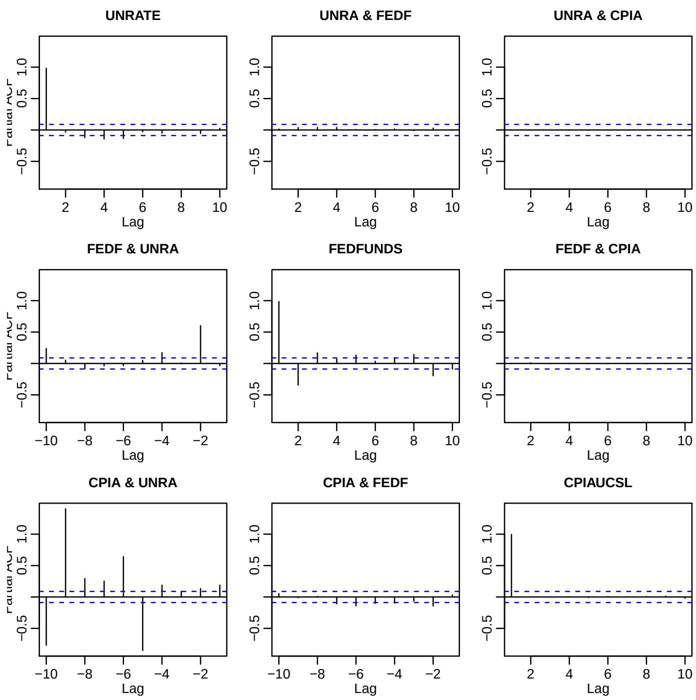
```
> # Consider focusing on 3 variables
> ymat0<-merge(UNRATE, FEDFUNDS, CPIAUCSL)
> ind.quarterly0<-1*(is.na(ymat0[,3])==FALSE)
> sum(ind.quarterly0)

[1] 800

> dim(ymat0)

[1] 801   3

> ymat00<-ymat0[which(ind.quarterly0==1),]

> head(ymat00)

           UNRATE FEDFUNDS CPIAUCSL
1947-01-01     NA       NA    21.48
1947-02-01     NA       NA    21.62
1947-03-01     NA       NA    22.00
1947-04-01     NA       NA    22.00
1947-05-01     NA       NA    21.95
1947-06-01     NA       NA    22.08

> par(mfcol=c(3,1))
> plot(ymat00[,1],main=dimnames(ymat00)[[2]][1])
> plot(ymat00[,2],main=dimnames(ymat00)[[2]][2])
> plot(ymat00[,3],main=dimnames(ymat00)[[2]][3])
```




```
> # Extract window from 1960-2000
>
>  ymat00.0<-window(ymat00,
+                  start = as.Date("1960-01-01"),
+                  end = as.Date("2000-12-31"))
> dim(ymat00.0)

[1] 492   3

> head(ymat00.0)

           UNRATE FEDFUNDS CPIAUCSL
1960-01-01    5.2     3.99    29.37
1960-02-01    4.8     3.97    29.41
1960-03-01    5.4     3.84    29.41
1960-04-01    5.2     3.92    29.54
1960-05-01    5.1     3.85    29.57
1960-06-01    5.4     3.32    29.61

> acf(ymat00.0, lag.max=10)
```




```
> acf(ymat00.0, type="partial", lag.max=10)
```



1.4 Vector Autoregressive (VAR) Model of Reduced Set

```
> # The function VARselect() is from the package vars; see Pfaff(2008).
> #   This function identifies the optimal VAR(p) order p.
> ymat00.0.VAR.const<-VARselect(ymat00.0, lag.max=12, type="const")
> # Print out the VAR order identified by different information criteria
> ymat00.0.VAR.const$selection

AIC(n)  HQ(n)  SC(n) FPE(n)
    12      5      2     12

> # Fit the VAR model corresponding to the Schwarz Criterion (SC) which is the BIC
> ymat00.0.VAR.const.0<-VAR(ymat00.0, p=ymat00.0.VAR.const$selection[3],type="const")
> options(show.signif.stars=FALSE)
```

> summary(ymat00.0.VAR.const.0)

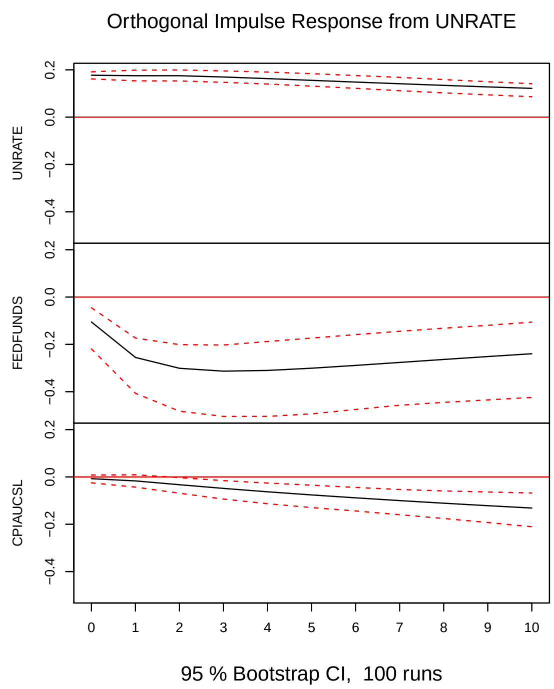
```
VAR Estimation Results:
=========================
Endogenous variables: UNRATE, FEDFUNDS, CPIAUCSL
Deterministic variables: const
Sample size: 490
Log Likelihood: -90.684
Roots of the characteristic polynomial:
1.002 0.9863 0.9524 0.4675 0.3314 0.08405
Call:
VAR(y = ymat00.0, p = ymat00.0.VAR.const$selection[3], type = "const")


Estimation results for equation UNRATE:
=======================================
UNRATE = UNRATE.l1 + FEDFUNDS.l1 + CPIAUCSL.l1 + UNRATE.l2 + FEDFUNDS.l2 + CPIAUCSL.l2 + con

            Estimate Std. Error t value Pr(>|t|)
UNRATE.l1    0.97239    0.04593  21.171  < 2e-16
FEDFUNDS.l1 -0.02928    0.01363  -2.148  0.03222
CPIAUCSL.l1  0.01744    0.04114   0.424  0.67176
UNRATE.l2    0.01157    0.04557   0.254  0.79974
FEDFUNDS.l2  0.04348    0.01373   3.168  0.00163
CPIAUCSL.l2 -0.01777    0.04121  -0.431  0.66642
const        0.02390    0.03558   0.672  0.50212


Residual standard error: 0.177 on 483 degrees of freedom
Multiple R-Squared: 0.9865,        Adjusted R-squared: 0.9864
F-statistic:  5891 on 6 and 483 DF,  p-value: < 2.2e-16


Estimation results for equation FEDFUNDS:
=========================================
FEDFUNDS = UNRATE.l1 + FEDFUNDS.l1 + CPIAUCSL.l1 + UNRATE.l2 + FEDFUNDS.l2 + CPIAUCSL.l2 + c

            Estimate Std. Error t value Pr(>|t|)
UNRATE.l1   -0.65364    0.14326  -4.563 6.42e-06
FEDFUNDS.l1  1.31042    0.04252  30.816  < 2e-16
CPIAUCSL.l1  0.20253    0.12832   1.578   0.1152
UNRATE.l2    0.64608    0.14213   4.546 6.93e-06
FEDFUNDS.l2 -0.33631    0.04281  -7.856 2.60e-14
CPIAUCSL.l2 -0.20311    0.12854  -1.580   0.1147
const        0.20704    0.11098   1.866   0.0627
```

```
Residual standard error: 0.5521 on 483 degrees of freedom
Multiple R-Squared: 0.9709,	Adjusted R-squared: 0.9706
F-statistic:  2689 on 6 and 483 DF,  p-value: < 2.2e-16


Estimation results for equation CPIAUCSL:
=========================================
CPIAUCSL = UNRATE.l1 + FEDFUNDS.l1 + CPIAUCSL.l1 + UNRATE.l2 + FEDFUNDS.l2 + CPIAUCSL.l2 + c

              Estimate Std. Error t value Pr(>|t|)
UNRATE.l1    0.0003679  0.0473065   0.008   0.9938
FEDFUNDS.l1  0.0647816  0.0140419   4.613 5.08e-06
CPIAUCSL.l1  1.3723826  0.0423743  32.387  < 2e-16
UNRATE.l2   -0.0010824  0.0469318  -0.023   0.9816
FEDFUNDS.l2 -0.0395123  0.0141368  -2.795   0.0054
CPIAUCSL.l2 -0.3713534  0.0424442  -8.749  < 2e-16
const       -0.0654237  0.0366463  -1.785   0.0748


Residual standard error: 0.1823 on 483 degrees of freedom
Multiple R-Squared:     1,	Adjusted R-squared:     1
F-statistic: 5.755e+06 on 6 and 483 DF,  p-value: < 2.2e-16



Covariance matrix of residuals:
            UNRATE  FEDFUNDS  CPIAUCSL
UNRATE    0.031327 -0.018662 -0.001303
FEDFUNDS -0.018662  0.304767  0.008291
CPIAUCSL -0.001303  0.008291  0.033232

Correlation matrix of residuals:
           UNRATE FEDFUNDS CPIAUCSL
UNRATE    1.00000 -0.19099 -0.04038
FEDFUNDS -0.19099  1.00000  0.08239
CPIAUCSL -0.04038  0.08239  1.00000
```

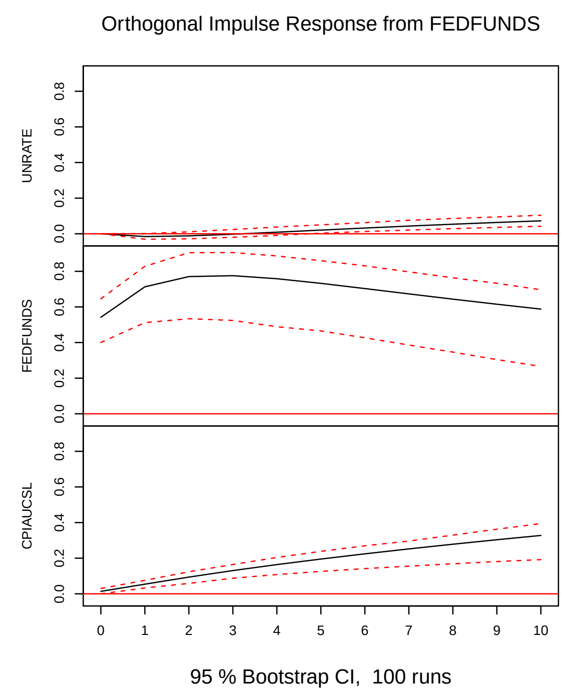
1.5 Impulse Response Functions for a Fitted VAR(p) Model

The impulse response function measure the impact of a unit innovation (impulse) in a given variable on all the dependent variables in the VAR model.

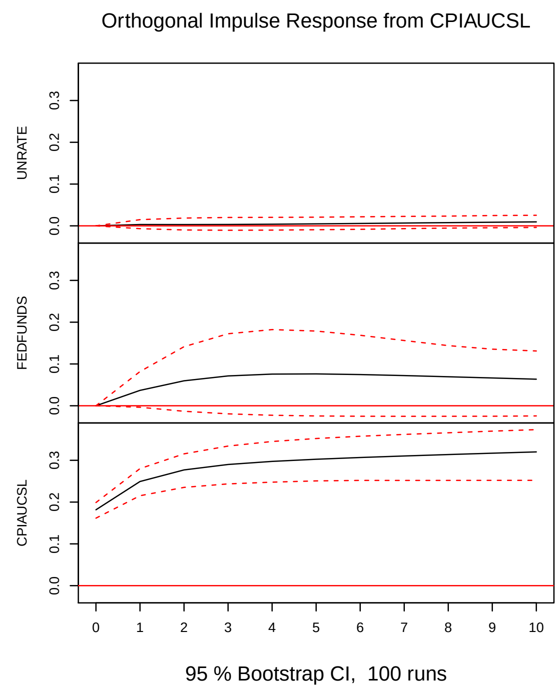
```
> plot(irf(ymat00.0.VAR.const.0, impulse="UNRATE"))
>
> # When unemployment rises:
> #    the Federal Funds rate is projected to decline
> #    (consistent with Federal Reserve Policy)
> #
> #    the CPI decreases (lower employment results in less
> #        pressure to increase consumer prices)
```



```
> plot(irf(ymat00.0.VAR.const.0, impulse="FEDFUNDS"))
>
> # When the Fed Funds rate increases:
> #
> #   The Unemployment rate tends to increase;
> #   so reducing the Fed Funds rate would tend to reduce unemployment
>
> #   The CPI increases; increases in the Fed Funds rate are
> #   associated with increase in CPI over future quarters
```

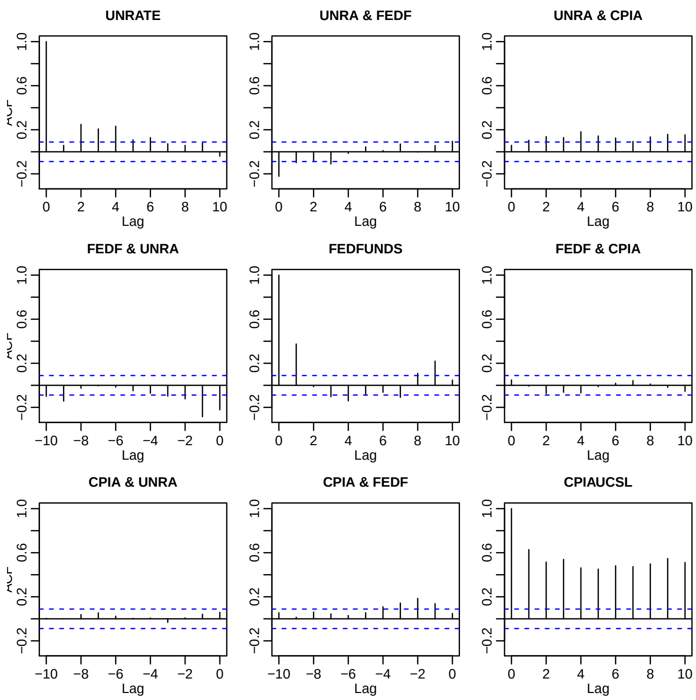


```
> plot(irf(ymat00.0.VAR.const.0, impulse="CPIAUCSL"))
>
> # When the CPI increases
> #
> #   The Federal Funds rate tends to increase over subsequent quarters.
> #   This is consistent with Federal Reserve policy of raising
> #   interest rates to control for inflation.
```

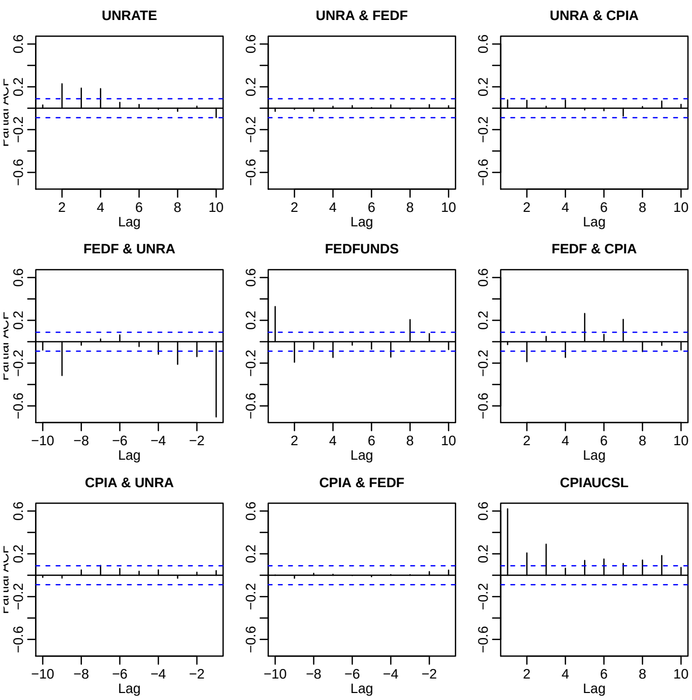


1.6 Ordinary and Partial Autocorrelations of Differenced Series

```
> ymat000.0<-na.omit(diff(ymat00.0))
> acf(ymat000.0, lag.max=10)
```




```
> acf(ymat000.0, type="partial", lag.max=10)
```



1.7 Vector Autoregressive (VAR) Model with Differenced Series

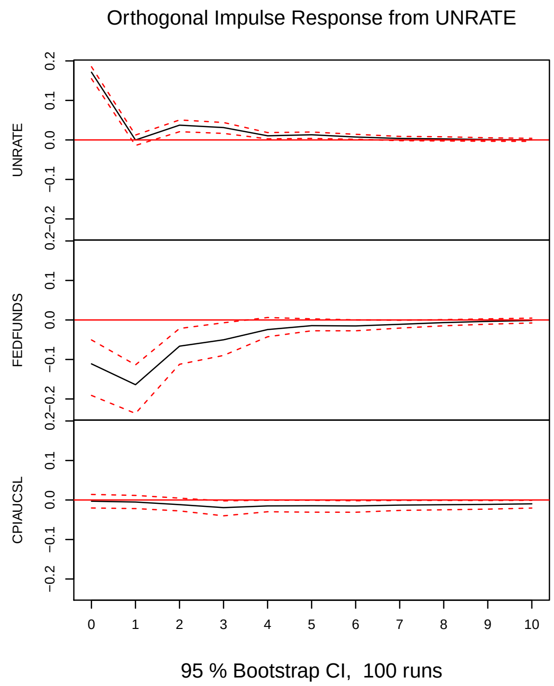
```
> # The function VARselect() is from the package vars; see Pfaff(2008).
> #   This function identifies the optimal VAR(p) order p.
> ymat000.0.VAR.const<-VARselect(ymat000.0, lag.max=12, type="const")
> # Print out the VAR order identified by different information criteria
> ymat000.0.VAR.const$selection

AIC(n)  HQ(n)  SC(n) FPE(n)
    12      3      3     12

> # Fit the VAR model corresponding to the Schwarz Criterion (SC) which is the BIC
> ymat000.0.VAR.const.0<-VAR(ymat000.0, p=ymat000.0.VAR.const$selection[3],type="const")
> options(show.signif.stars=FALSE)
```

```
> summary(ymat000.0.VAR.const.0)

VAR Estimation Results:
=========================
Endogenous variables: UNRATE, FEDFUNDS, CPIAUCSL
Deterministic variables: const
Sample size: 488
Log Likelihood: -69.438
Roots of the characteristic polynomial:
0.8369 0.7659 0.584 0.584 0.5755 0.5755 0.4907 0.4907 0.3088
Call:
VAR(y = ymat000.0, p = ymat000.0.VAR.const$selection[3], type = "const")


Estimation results for equation UNRATE:
=======================================
UNRATE = UNRATE.l1 + FEDFUNDS.l1 + CPIAUCSL.l1 + UNRATE.l2 + FEDFUNDS.l2 + CPIAUCSL.l2 + UNR

             Estimate Std. Error t value Pr(>|t|)
UNRATE.l1   -0.007647   0.045642  -0.168   0.8670
FEDFUNDS.l1 -0.010946   0.014641  -0.748   0.4551
CPIAUCSL.l1  0.033734   0.040703   0.829   0.4076
UNRATE.l2    0.220669   0.044850   4.920 1.19e-06
FEDFUNDS.l2  0.016837   0.015397   1.094   0.2747
CPIAUCSL.l2  0.060812   0.044099   1.379   0.1685
UNRATE.l3    0.182936   0.045599   4.012 6.99e-05
FEDFUNDS.l3 -0.027506   0.014294  -1.924   0.0549
CPIAUCSL.l3  0.015690   0.040408   0.388   0.6980
const       -0.034330   0.013372  -2.567   0.0106


Residual standard error: 0.1714 on 478 degrees of freedom
Multiple R-Squared: 0.1238,        Adjusted R-squared: 0.1073
F-statistic: 7.507 on 9 and 478 DF,  p-value: 2.636e-10


Estimation results for equation FEDFUNDS:
=========================================
FEDFUNDS = UNRATE.l1 + FEDFUNDS.l1 + CPIAUCSL.l1 + UNRATE.l2 + FEDFUNDS.l2 + CPIAUCSL.l2 + U

             Estimate Std. Error t value Pr(>|t|)
UNRATE.l1   -0.712680   0.145160  -4.910 1.25e-06
FEDFUNDS.l1  0.371252   0.046564   7.973 1.15e-14
CPIAUCSL.l1  0.160947   0.129450   1.243 0.214364
UNRATE.l2   -0.147333   0.142641  -1.033 0.302175
FEDFUNDS.l2 -0.179049   0.048968  -3.656 0.000284
```

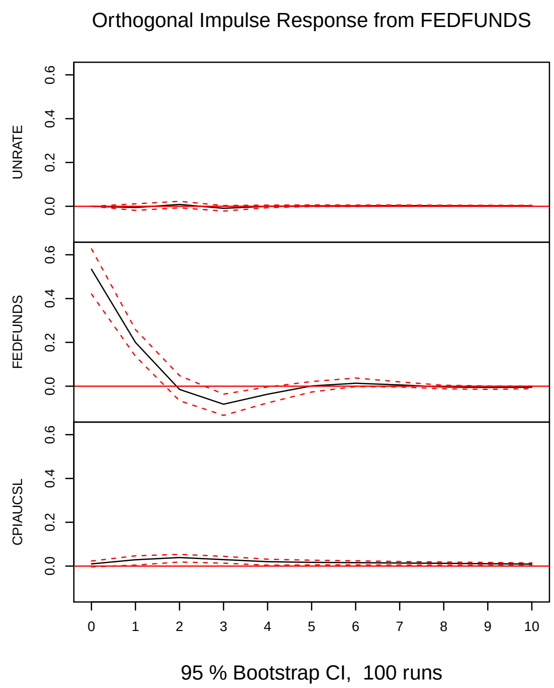
```
CPIAUCSL.l2 -0.189085   0.140252  -1.348 0.178238
UNRATE.l3   -0.204277   0.145021  -1.409 0.159604
FEDFUNDS.l3 -0.069421   0.045461  -1.527 0.127407
CPIAUCSL.l3  0.046866   0.128511   0.365 0.715507
const       -0.004283   0.042528  -0.101 0.919823


Residual standard error: 0.5451 on 478 degrees of freedom
Multiple R-Squared: 0.2256,        Adjusted R-squared: 0.211
F-statistic: 15.47 on 9 and 478 DF,  p-value: < 2.2e-16


Estimation results for equation CPIAUCSL:
=========================================
CPIAUCSL = UNRATE.l1 + FEDFUNDS.l1 + CPIAUCSL.l1 + UNRATE.l2 + FEDFUNDS.l2 + CPIAUCSL.l2 + U

             Estimate Std. Error t value Pr(>|t|)
UNRATE.l1    0.007148   0.049135   0.145  0.88439
FEDFUNDS.l1  0.046389   0.015762   2.943  0.00341
CPIAUCSL.l1  0.415128   0.043818   9.474  < 2e-16
UNRATE.l2    0.010148   0.048283   0.210  0.83361
FEDFUNDS.l2  0.032147   0.016575   1.939  0.05303
CPIAUCSL.l2  0.067344   0.047474   1.419  0.15668
UNRATE.l3   -0.026752   0.049088  -0.545  0.58603
FEDFUNDS.l3  0.005058   0.015388   0.329  0.74252
CPIAUCSL.l3  0.291014   0.043500   6.690 6.26e-11
const        0.067658   0.014395   4.700 3.41e-06


Residual standard error: 0.1845 on 478 degrees of freedom
Multiple R-Squared: 0.4855,        Adjusted R-squared: 0.4758
F-statistic: 50.11 on 9 and 478 DF,  p-value: < 2.2e-16



Covariance matrix of residuals:
             UNRATE  FEDFUNDS   CPIAUCSL
UNRATE    0.0293761 -0.019046 -0.0005205
FEDFUNDS -0.0190462  0.297133  0.0057060
CPIAUCSL -0.0005205  0.005706  0.0340444

Correlation matrix of residuals:
           UNRATE FEDFUNDS CPIAUCSL
UNRATE    1.00000 -0.20386 -0.01646
FEDFUNDS -0.20386  1.00000  0.05673
CPIAUCSL -0.01646  0.05673  1.00000
```

1.8 Impulse Response Functions for VAR(p) Fit of Differenced Series

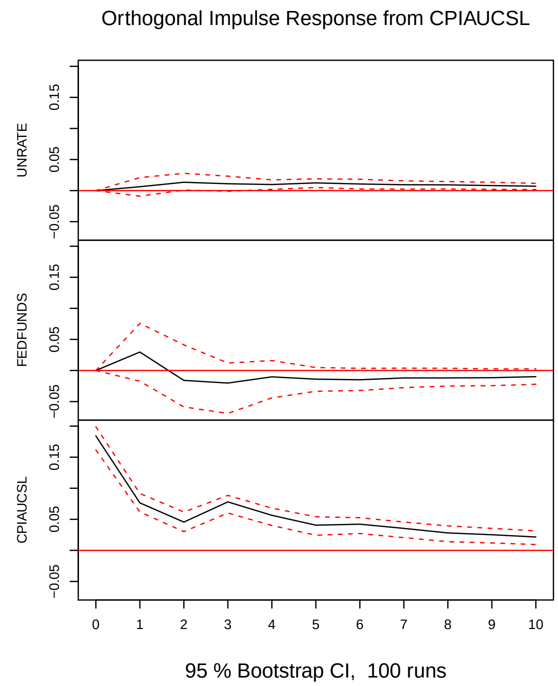
```
> plot(irf(ymat000.0.VAR.const.0, impulse="UNRATE"))
```



```
> plot(irf(ymat000.0.VAR.const.0, impulse="FEDFUNDS"))
>
```



```
> plot(irf(ymat000.0.VAR.const.0, impulse="CPIAUCSL"))
```



Interpreting the impulse response functions for the VAR model of the differenced series, we note:

- When unemployment increases, the Fed Funds rate tends to decrease over subsequent quarters, consistent with Federal Reserve policies (i.e., stimulating economic growth and employment with lower interest rates).
- When the Fed Funds rate increases, there is a modest increase in inflation (CPIA). This is consistent with the Fed raising rates to control inflation which tends to persist for several quarters (note the high 3-rd quarter lag partial autocorrelation in CPIAUCSL).
- When inflation (CPIAUCSL) increases, unemployment tends to rise modestly, and the Fed Funds rate tends to increase.

References

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