18.950 Differential Geometry
Fall 2008

18.950 Homework 2

1. (5 points) Let $c : I \to \mathbb{R}^2$ be a regular curve, whose curvature satisfies $|\kappa_c| \leq 1$ everywhere. Now stretch it in one direction, defining $d(t) = (2c_1(t), c_2(t))$. What can one say about κ_d? Obviously, the circle is the first example to think about here.

2. (15 points) For this problem, you'll need elementary knowledge of the complex numbers, at least knowing how to do arithmetic operations on $\mathbb{C} = \mathbb{R}^2$. You'll also need MAPLE or Mathematica. On the positive side, you don't need to give proofs of your insights.

Let p be a polynomial, for instance $p(z) = z^4/4 - 8/3z^3 + 17/2z^2 - 10z$, $p(z) = z^4 - z$, or $p(z) = z^4/4 + z^3/3 + z^2/2 + z$. Fix some radius R and take the closed 2π-periodic curve $c(t) = p(R\exp(it))$. How does the total curvature vary with R? Where are the jumps and how many are there? Let the computer draw some pictures for you.