LECTURE 14: PROOF OF HUREWICZ

Proposition 0.1. The Hurewicz homomorphism induces a map of long exact sequences

$$\begin{array}{ccccccccc} \cdots & \longrightarrow & \pi_k(A) & \longrightarrow & \pi_k(X) & \longrightarrow & \pi_k(X,A) & \longrightarrow & \cdots \\ & & \downarrow & & \downarrow & & \downarrow & & \\ \cdots & \longrightarrow & \widetilde{H}_k(A) & \longrightarrow & \widetilde{H}_k(X) & \longrightarrow & H_k(X,A) & \longrightarrow & \cdots \end{array}$$

Let X be an $(m-1)$-connected CW-complex. The Hurewicz theorem is proved as follows:

(1) Use homotopy excision, and the homework problem on the split short exact sequence of the homotopy of a wedge, to show that the Hurewicz theorem holds for arbitrary wedges of spheres.
(2) Use the argument for cellular approximation to show that X is homotopy equivalence to a CW complex Y with no cells below dimension m.
(3) Apply Proposition 0.1 to perform induction on the skeletal filtration of Y, using homotopy excision and the case of spheres to handle the relative homotopy groups.

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