18.905 Problem Set 12 (Final)

Due Friday, December 8 in class

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1. Suppose $f \in C^p(X)$, $g \in C^q(X)$, and $x \in C_{p+q+r}(X)$. Show, using the definition of the cap product given in class, that
$$g \frown (f \frown x) = (f \smile g) \frown x \in C_r(X).$$
Show that this makes $C_*(X) = \oplus_n C_n(X)$ into a *right* module over the ring $C^*(X) = \oplus_n C^n(X)$. (For this reason, Hatcher writes cap products with the terms reversed.)

2. Hatcher, exercise 17 on page 259.

3. Hatcher, exercise 22 on page 259.

4. Suppose that M is a compact orientable manifold, and let ΣM be its suspension
$$M \times [0,1]/\{(x,0) \sim (y,0), (x,1) \sim (y,1)\}.$$
Show that ΣM cannot be a manifold unless M has the same homology as a sphere. (You may not assume that M is connected.)