A project laboratory in mathematics

New Directions for Mathematics REUs
Mt Holyoke
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18.821 Project Laboratory in Mathematics

Initiated in Spring 2004
Just finished 17th version
332 students, 9 faculty over 9.5 years

Main objective: provide students with some experience with the research process in mathematics :
-- some hint of the mystery, frustration, and exhilaration of the research experience : and it's something they can can do!
-- teamwork
-- written and oral presentation

Class Characteristics

Students: Math majors, mainly juniors and seniors.
Capped at 27 students.

Prerequisites: At least two math subjects beyond the basics.

Satisfies: Undergraduate Lab and Writing requirements.
(Both can be satisfied in other ways.)

Staffing: Faculty lead, two postdocs or grad students
and help from a communications expert.

Available: to all math majors (but not required).

Course Work Cycle (in 13 week term)

-- Staff works with students; be sincere but not prescriptive; give feedback on progress. (3 weeks)

-- Students write and submit first draft. Select next project.

-- Staff reads and marks up first draft. (3 days)

-- Debriefing with mentor and course leader (40 minutes)

-- Students revise and resubmit (1 week)

Other class components

-- Introductory lecture

-- Workshops:
  -- Team building
  -- Writing
  -- Presenting

-- Class presentations: each team on one project.
  -- Other class members fill out comment sheets
  -- Preceded by practice presentation (leave 2 hours)

The projects

Analysis 9
Combinatorics 7
Probability 6
Dynamical systems 4
Algebra 4
Number theory 3
Geometry 2

Generated over the years by MIT faculty; project list is a work in progress.

Key feature: Can be taken in various directions; extensible. Normally not “new” research!

Example -- Attraction (dynamical systems)

This problem asks you explore a deceptively simple dynamical system and discover some of surprising properties. Consider the motion of four particles A,B,C, and D in the plane. The particles start at four random points in the plane. Each particle moves with unit speed. A moves towards B, B towards C, C towards D and D towards A. What happens (qualitatively), and how (quantitatively, in terms of, say, angles and log distances)?

The simplest case is when the starting positions form a square. Actually, the game with three particles, but with various starting positions, is already quite interesting. Other generalizations would be to play the game on a sphere or in higher-dimensional spaces.

An Attractive Dynamical System

G. Statev, A. Wang

April 21, 2013

Abstract

We investigate a dynamical system consisting of n point in which every point moves towards some other point with unit velocity. We prove that for every initial configuration of points, at least two of them will collide in a finite amount of time. We show that its possible for all n points to collide at the same time but we suspect that this need not always be true. We give a limiting case configuration in which the statement is indeed not true.

1 Introduction

In this paper we will undertake an investigation of the following dynamical system: Consider n distinct points $A_1, A_2, \ldots, A_n$ in space. Each point moves with unit velocity towards another point. Specifically, point A_1 moves towards point A_2, point A_2 moves towards A_3,..., point A_{n-1} moves towards point A_n and point A_n moves towards point A_1. If we denote by $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$ the position vectors of these points in Cartesian coordinates, then the system can be described by the following set of non-linear ordinary differential equations:

$$\begin{aligned}
\frac{d\vec{r}_1}{dt} &= \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}, \\
\frac{d\vec{r}_2}{dt} &= \frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|}, \\
&\vdots \\
\frac{d\vec{r}_{n-1}}{dt} &= \frac{\vec{r}_n - \vec{r}_{n-1}}{|\vec{r}_n - \vec{r}_{n-1}|},
\end{aligned} \tag{1}$$

Student feedback

Student A - This was great. I wish I had had this before my REU last summer.

Student B - I was very, very surprised by how much I enjoyed this course!

Student C - I think 821 is a great subject to take for the math CI-M requirement and the feedback on the papers is incredibly useful to learn how to refine your writing for these technical topics. Furthermore giving the presentation and receiving feedback from the professor and all of the students is incredibly useful.

A reproducible experiment

Variations of this can work in many settings. It has been given at Berkeley and at the University of Michigan (under Jeff Lagarias).

On the web: http://mathcomm.org/courses/math-lab-class/

Coming soon: An OpenCourseWare representation.

Invitation: Try it yourself. I'll give you our project list. In exchange, give us projects that you generate!

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