Hints for Problem Set 3

- *Problem 1e*
  Given a matrix M, how can you make a new matrix N such that the largest eigenvalue of N is related to the smallest eigenvalue of M?

- *Problem 1f*
  Try to generalize the idea from 1e. Given a matrix M, how can you create a new matrix N so that the biggest eigenvalue of N is related to the eigenvalue of M closest to lambda?

- *Problem 2b*
  If I have a polynomial p(x) in one variable and I apply it to a matrix with the given eigenvalue bounds, I get a new matrix N. What can I say about its eigenvalues? What properties of p will guarantee that N still has the same eigenvector with eigenvalue 1 but that every vector orthogonal to it gets shrunk significantly in norm?

- *Problem 2c*
  Look really carefully at the description of conjugate gradient in Shewchuk's article. I claim that we've already computed everything I'm asking you for.

- *Problem 2e*
  For any polynomial p of degree t-1, show that there is a vector v in K(A,x,t) such that v=p(A)x. Argue that for a random x, there is some polynomial p for which the corresponding v gives a good Rayleigh quotient, and thus a good estimate for the biggest and smallest eigenvalues of A.

- *Problem 3*
  Make a new 2n x 2n matrix A and a vector q, and solve the linear system:
  Ax = q

  q will be the vector [b; -b].
  A will be made (somehow) by combining (somehow—maybe you'll need to add them to each other, etc.) the following pieces:

  D = the diagonal of M
  P = the matrix of positive entries of M (with everything else set to zero)
  N = the matrix of negative entries of M (with everything else set to zero)

- *Problem 4d*
  You have to show how to route the edges of G over those of H' with low maximum total stretch, so that you can apply part (a). Part (c) tells you how to route the edges that are internal to a give G_i. Use this to get a bound on the contribution of these edges to the total stretch.

  You then just need to figure out how to route the edges that cross from one G_i to some other G_i. Route these edges internally in G_i, across the bridge edge we've added, and then internally in G_j. Bound the contribution of these edges.

18.409 Topics in Theoretical Computer Science: An Algorithmist's Toolkit
Fall 2009