18.335 Midterm. November 3, 2004

Name:

Problem 1	
Problem 2	
Problem 3	
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Problem 5	
Problem 6	
Total	

In all problems, all matrices are real and square and all vectors are real.

1. (5 points) Assume (do not prove here)

$$\|x\|_\infty \leq \|x\|_2 \leq \sqrt{n}\|x\|_\infty, \text{ for all } x \in \mathbf{R}^n.$$

   Show that for any matrix A

$$\|A\|_\infty \leq \sqrt{n}\|A\|_2 \leq n\|A\|_\infty.$$

2. (5 points) Let A be symmetric positive definite matrix with Cholesky factor C, i.e. $A = C^T C$. Show that $\|A\|_2 = \|C\|_2^2$ and that $\kappa_2(A) = (\kappa_2(C))^2$, where $\kappa_2(X)$ is the condition number of the matrix X measured in the two-norm.

3. A matrix A is called strictly column diagonally dominant if $|a_{ii}| > \sum_{j \neq i} |a_{ji}|$ for all i.

   (a) (5 points) Show that such an A is nonsingular.

   (b) (5 points) Show that no pivoting is needed when computing $A = LU$. In other words, if we did do partial pivoting to compute $PA = LU$, P a permutation matrix, then we would get $P = I$. Hint: Show that after one step of Gaussian elimination, the bottom right $n-1$ by $n-1$ submatrix is also strictly column diagonally dominant.

4. (5 points) Let A be skew Hermitian, i.e. $A^* = -A$. Prove that the eigenvalues of A are purely imaginary and that $I - A$ is nonsingular.

5. (5 points) Let $\|\cdot\|$ be an operator norm. Prove that if $\|A\| < 1$ then $I - A$ is invertible.

6. (5 points) Let $x = (1, 2, 3, 4, 5, 6, 7)^T$ and $y = (-7.5, 2, -4, -4, 2, 3, 0.5)^T$. In double precision IEEE binary floating point arithmetic, true or false:

$$\text{fl}(x^T y) = 0?$$

   Explain.