Homework: 9

Due December 1.

1. A is a square matrix of size 19,000. All entries of A are zero except for the primes 2,3,5,7,...,212369 along the main diagonal and the number 1 in all the positions a_{ij} with |i-j|=1,2,4,8,...,16384. Compute the (1,1) entry of A^{-1}.
   Solution: See this website, and problem 7 (and its solution) there.
2. The 200-by-200 (diagonally dominant) matrix A has offdiagonal entries -1/i-1/j and row sums s(i)=sum(A(i,:))=2^{2i}. Compute the smallest in magnitude eigenvalue of A. (Hint: The diagonal entries A(i,i) can then be computed as sum of positive (why?) numbers, thus to high relative accuracy. Abandoning the row sums s(i), however, robs you of any chance of computing the smallest eigenvalue accurately). You will encounter no (true) subtractions when running Gaussian elimination, obtaining the LU decomposition of A to high relative accuracy componentwise. Using the LU factors to compute the inverse of A one column at a time will not result in any subtractions either, yielding a positive A^{-1}.)
   Solution: mmatinverse.pdf, hw6.m, InverseMM.m.
   Answer: 1.207993236710136e+002