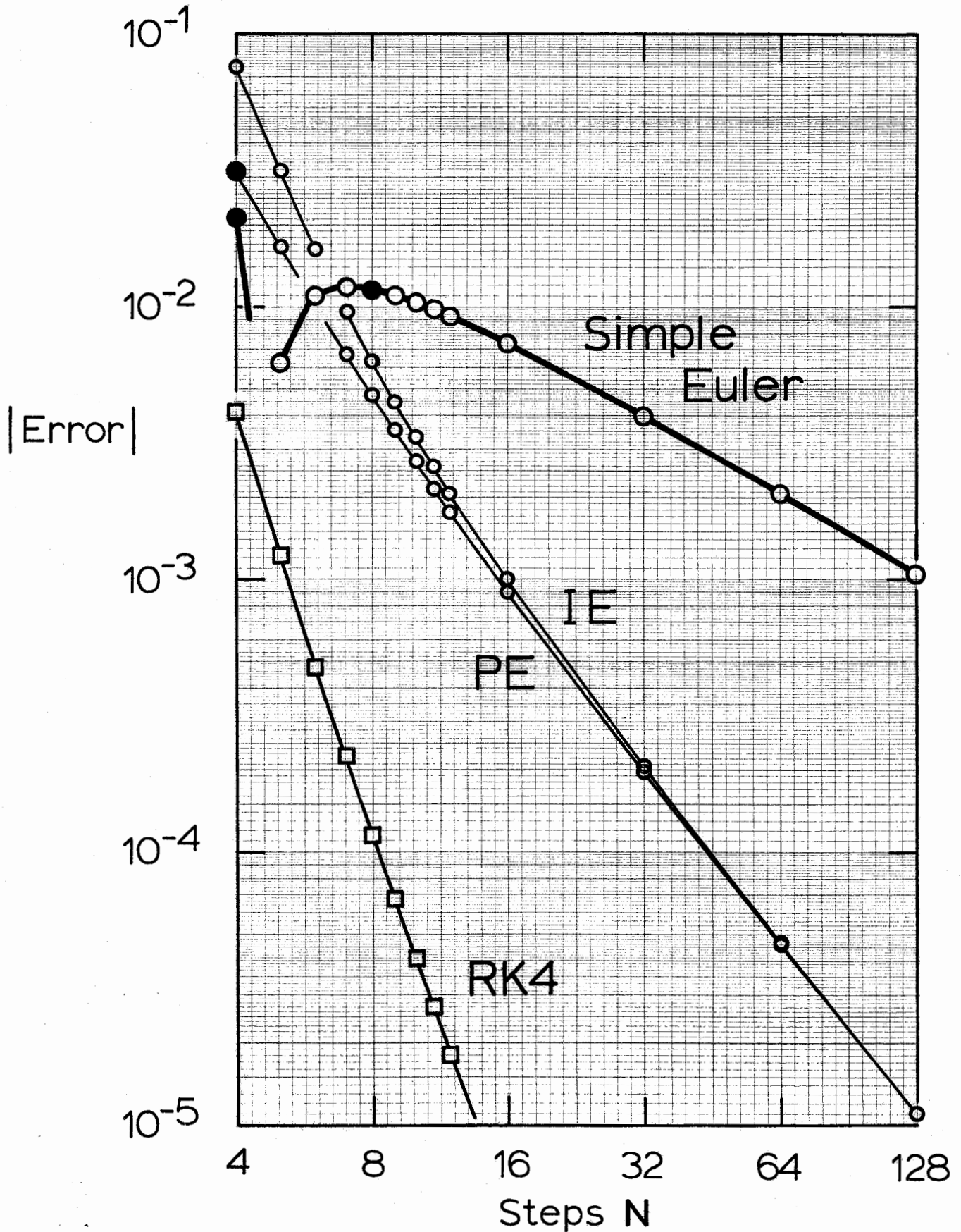


18.03 Rates of Convergence*

February 21, 1990

* of integration schemes struggling with $y' = 1 - xy^2$, $y(0) = 0$, $y(2) = ?$



Here are various estimates of $y(2)$ again ... in more detail, and with errors shown explicitly:

Simple Euler

Parabolic Euler

Improved Euler

4th-order Runge-Kutta

Steps	error		error		error		error	
N = 4	0.7509 74	-212 38	0.7409 36	-312 76	0.6960 17	-761 95	0.7680 82	-41 30
5	7784 54	+62 42	7550 70	-166 42	7406 11	-316 01	7709 92	-12 20
6	7831 87	+109 75	7621 33	-100 79	7559 95	-162 17	7717 36	-4 76
7	7839 75	+117 63	7655 30	-66 82	7625 72	-96 40	7719 90	-2 22
8	0.7837 60	+115 48	0.7674 83	-47 29	0.7658 75	-63 37	0.7720 96	-1 16
9	7832 34	110 22	7686 99	-35 13	7677 42	-44 70	7721 45	-67
10	7826 25	104 13	7695 03	-27 09	7688 93	-33 19	7721 71	-41
11	7820 19	98 07	7700 61	-21 51	7696 51	-25 61	7721 85	-27
12	0.7814 46	92 34	0.7704 63	-17 49	0.7701 75	-20 37	0.7721 94	-18
16	0.7795 95	73 83	0.7713 13	-8 99	0.7712 18	-9 94	0.7722 07	-5
32	7761 93	39 81	7720 15	-1 97	7720 06	-2 06	7722 12	0
64	7742 65	20 53	7721 66	-46	7721 65	-47	7722 12	0
N = 128	0.7732 53	+10 41	0.7722 01	-11	0.7722 01	-11	0.7722 12	0

Order m = 1

Order m = 2

Order m = 4