18.330 Problem Set 7

Due in class: Fri 02 Apr 04

19 For the initial-value ODE problem $dy/dx = x^2 - y^2$, $y(0) = 0$:

(a) Proceed to estimate $y(2)$ via a sequence of **simple Euler** integrations using steps of size $h = 1, 1/2, 1/4, 1/8, \ldots$ and as much Richardson extrapolation as you can stomach.

(b) Repeat the above using some well-known **second-order** scheme.

(c) Recalculate $y(2)$ via **Taylor series** for the function $u(x)$, after making the Riccati substitution $y(x) = u'(x) / u(x)$.

(d) Quickly estimate $y(1000)$ to at least SIX significant digits.

20 By any reasonably elegant and efficient strategy, try and answer to high accuracy:

(a) If $y(0) = 0$ and $dy/dx = e^{-xy}$, what **limiting value** does the solution $y(x)$ approach as x grows large and positive?

(b) If again $y(0) = 0$ but $dy/dx = e^{+xy}$, at what finite value of x does this new $y(x)$ "explode" upward to +infinity?

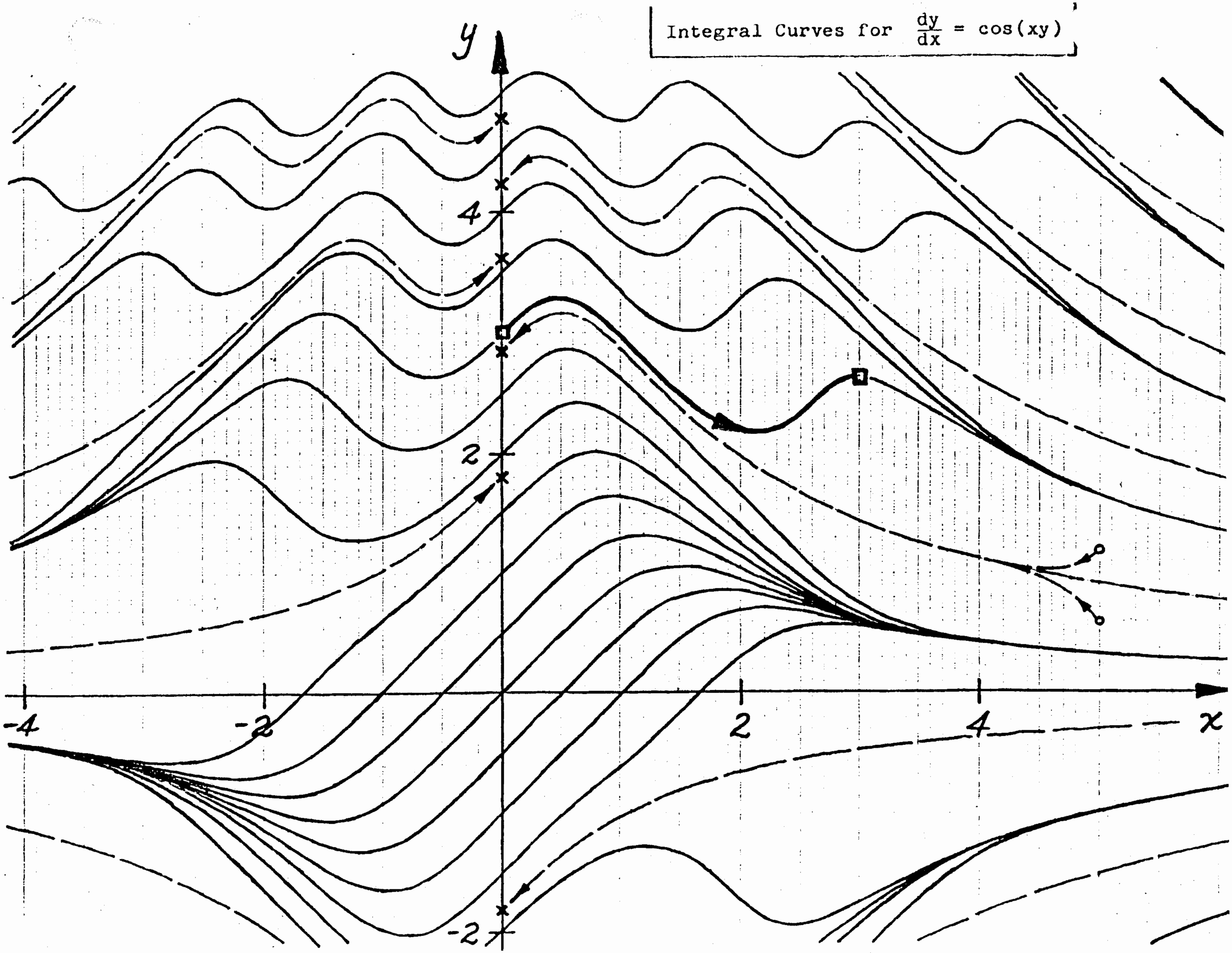
21 Shown overleaf are some attractive integral curves for the ODE

$$y' = \cos(xy) \quad .$$

Find to high accuracy (again at least 6 and preferably 9+ decimals)

(a) the value $y(3)$, given that $y(0) = 3$; and

(b) that **critical starting value** $y(0)$ = approx 2.8 — marked with one of the x'es — which like some Continental Divide separates the first and second solution bundles here.



Integral Curves for $\frac{dy}{dx} = \cos(xy)$
y
4
2
-2
-4
-2
2
4
x