18.330 Problem Set 2 Due in class: Fri 20 Feb 04

(4) Make it your business to track down to high accuracy, in two ways starting from the initial guess $z_0 = 1 + i$, that root of

$$z^4 + z + 1 = 0$$

which resides in the first quadrant of the complex z-plane. Employ

(a) the **complex Newton** method , and less efficiently also

(b) some **real variable** search for that x,y pair which solves simultaneously the related pair of equations

$$x^4 - 6x^2y^2 + y^4 + x + 1 = 0 , \quad 4x^3y - 4xy^3 + y = 0 .$$

(5) The well-known Legendre polynomials

$P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2-1)/2$, $P_3(x) = (5x^3-3x)/2$, ...

obey the **recurrence relation**

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x) .$$

Hence locate via Newton's method to at least 6 decimals that root of $P_{30}(x)$ which lies closest to $x = 0.5$, left or right.

(6) As implied by the diagram overleaf, the famous **quadratic** iteration

$$x_{n+1} = Cx_n(1-x_n) \equiv g(x_n)$$

settles down to a stable two-hop cycle when the constant C exceeds 3 but does not exceed an upper critical value roughly equal to 3.45 .

Your task: Analyze the stability of the "**stroboscopic**" iteration

$$x_{n+2} = g[g(x_n)] \equiv G(x_n) ,$$

and thereby locate the precise value of C at which this related two-hop cycle bifurcates in turn.

Results of the iteration

$$x_{n+1} = C\, x_n (1 - x_n)$$