Waves and Imaging
Class notes - 18.325

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Preface

In the margins of this text we use

- the symbol (!) to draw attention when a physical assumption or simplification is made; and
- the symbol ($) to draw attention when a mathematical fact is stated without proof.

Thanks are extended to the following people for discussions, suggestions, and contributions to early drafts: William Symes, Thibaut Lienart, Nicholas Maxwell, Pierre-David Letourneau, Russell Hewett, and Vincent Jugnon.

These notes are accompanied by computer exercises in Python, that show how to code the adjoint-state method in 1D, in a step-by-step fashion, from scratch. They are provided by Russell Hewett, as part of our software platform, the Python Seismic Imaging Toolbox (PySIT), available at http://pysit.org.

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