More than 1-D. Conservation laws in 2-D or 3-D.
Flux q is now a vector. Use Gauss theorem to obtain general form $\rho_t + \text{div}(q) = S$ (S equal sources & sinks).

Examples of S:

- Cars flowing in/out of highway through commuter township.
- Water flowing into river from small affluents.
- Heating by electromagnetic waves [microwave oven].

EXAMPLE: Heat flow in 2-D or 3-D.

Then,
$\rho = r\, c_v\, T$ = conserved stuff (heat) per unit mass

Where:
$c_v$ = specific heat of material
r = mass density
T = temperature
Fick's law applies:
Q = - kappa gradient(T), and kappa = heat conductivity.
Thus Heat equation:
$T_t$ = nu Laplacian T,
Where
nu = kappa/(r $c_v$) = coeff. thermal diffusion.

EXAMPLE:
Diffusion equation (Salt in water, sugar in coffee, ink in water, ETC.)
Same as heat equation
$C_t$ = nu*Laplacian C
Where
C = concentration (salt, ink, sugar, ...)
nu = diffusion coefficient.

intuitive justification of Fick's law.
DIMENSIONS and DIMENSIONAL ANALYSIS.
What are the dimensions of kappa? nu? $c_v$?

How long does it take sugar to sweeten a coffee cup without stirring? Idealized problem: start with a very small blob of ink, and ask:
What is the radius of the blob of ink, R = R(t), as the blob expands due to diffusion?
Dimensional analysis says: $R(t) \propto \sqrt{(nu*t)}$

In particular, let L be the size of the coffee cup. Sugar will reach whole cup when R = O(L) ===> time = O($L^2$/nu).
Also the relevant time needed to cool/heat a size L vessel.

These times are very long when measured in human-relevant scales. Hence stirring needed. Boiling and convection speed up heating.

Questions:

- why does stirring help?
- why does convection occur?
- what would happen when heating something with a flame in the absence of gravity?

At room temperature, in cm^2/sec

Thermal diffusivity: water ~ 0.0014 mercury ~ 0.042
Diffusion in water: NaCl ~ 10^{-5}

18.311 Principles of Applied Mathematics
Spring 2014