Do FD schemes. Follow the notes attached to PSet#08, plus examples.
Start with lecture in MATLAB® toolkit GBNS_lecture.

Further material in the web page [notes section] under
- Stability of Numerical Schemes for PDE's (Quick preview).
- Stability of Numerical Schemes for PDE's.
- Various lecture notes for 18311.

Section: Convergence of numerical Schemes.

Finite difference schemes for PDE's. Follow notes:

Stability of Numerical Schemes for PDE's.
1. Naive Scheme for the Wave Equation. USE the MATLAB script in the 18311 toolkit: GBNS_lecture.
2. von Neumann stability analysis for PDE's.
3. Numerical Viscosity and Stabilized Scheme.
4. Model equation.

1) von Neumann stability analysis.
Recall that solutions to time-evolution linear PDE's can be found by separation of the time variable --- recall analogy with ODE approach --- leading to an eigenvalue problem. Extend idea to constant coefficients linear FD schemes --- key to the von-Neuman stability analysis.

2) Examine instabilities using associated equation: Explain behavior via forward & backward heat equations.

3) Introduce stabilization by artificial viscosity (general idea). Relationship with the solutions of the heat equation by separation of variables exp(-k^2*t + i*k*x}.
Inspect what happens when a term u_{xx} is added to an equation like $u_t+u_x=0$. What does it do to the normal modes? Other examples [0 < D << 1]
- $u_t + u_{xxx} = D*u_{xx}$
- $u_t = v + D*u_{xx}$ and $v_t = u_{xx} + D*v_{xx}$

4) Define CONSISTENCY. von Neumann and consistency: Numerical and exact growth rates; comparison in the small k limit.

5) Define STABILITY
6) Lax Theorem: for linear schemes, Consistency+Stability ==> Convergence.

Further details:
7) Show that the general solution to a finite differences linear scheme can be written as a linear combination of the solutions G^j*e^(i*k*n) obtained by the von Neuman stability analysis. In other words, show that the matrix {w^{n*m}} has an inverse,

where $w = e^{i*2*\pi/N}$. This leads to the Discrete Fourier Transform (DFT), which will be the next course topic.

7) Associated equation. More examples of von Neuman stability analysis; associated equation, and stabilization by artificial viscosity: Lax-Friedrich sheme.

- Computer illustrations
- Numerical instabilities .... GBNS_lecture scheme.m
- Wave breaking and steepening .... demoWBRch_v02.m
- Convergence of Fourier Series .... fourierSC.m
- Talk about Gibb's phenomena [related to new topic below].

Further material [in the introduction to problem set #08]
Introduction to the vNSA problem series.

Example #1 in the notes
Explain idea behind "artificial viscosity" used to stabilize a scheme. Motivate it by looking at what adding a term u_{xx} to an equation like $u_t+u_x=0$ does to the normal modes. For that matter, what it does do $u_t+u_{xxx}=0$.

Forwards and backwards differences for u_t+u_x
Assigned in pset #08.

CFL condition. Necessary but not sufficient for stability
Both the "good" and "bad" schemes satisfy CFL if $dt < dx$.