18.311: Principles of Applied Mathematics Rodolfo Rosales
Lecture 24 Spring 2014

Compute time derivative of $\int \frac{1}{2}\rho^2 dx$ for viscous problem $\rho_t + (Q(\rho))_x = \nu*\rho_{xx}$
Show decreasing (dissipation).
Show time derivative does not go to zero (as ν vanishes) if there are shocks [plugin form ρ = R((x-s*t)/ν)].

RECALL/RECAP here: Information loss at shocks.

Gas Dynamics, Acoustics, and Strings

Gas Dynamics: Derive equations in 1-D (using conservation of mass and momentum), plus the quasi-equilibrium isentropic assumption p = p(ρ). Example: p = κ*ρ^γ
polytropic gas (ideal gas with constant specific heats).

Boundary conditions at the end of the pipe:

- Closed pipe u = 0
- Open pipe p = p_0

For smooth solutions, manipulate equations into the form
$\rho_t + u*\rho_x + \rho*u_x = 0$
$u_t + (a^2/\rho)*\rho_x + u*u_x = 0$
where $a^2 = dp/d\rho > 0$ [note that dp/dρ > 0].
Calculate a for ideal gas case: $a^2 = \gamma*p/\rho$.
Check dimensions: a is a velocity (sound speed, as we will see).

Derive also Shallow Water equations on a flat bottom. Simplify derivation by neglecting air pressure [makes no difference, since adding a constant to the pressure does not change the forces].
Note that these equations are THE SAME AS GAS DYNAMICS WITH γ = 2.
Note that √{dp/dρ} and √{g*h} are speeds.

For gas dynamics, at constant entropy, so that p = p(ρ):
Property of p(ρ): $a^2 = dp/d\rho > 0$.
Show a has dimensions of speed.
Compute what a is for Shallow Water.
Calculate a for tidal wave in the deep ocean (4.000 m).

Some rough numbers:
p = 1 atmosphere ~ 10^5 kg/(m*s^2) [about 10m of water depth].
ρ air ~ 1 kg/m^3 [about 1/1000 that of water].
γ air ~ 1.4
This yields a ~ 370 m/s [actual is ~ 340 m/s]

For shallow water: p = (1/2)*g*h^2, a = √(g*h).
4000 m depth yields: 200 m/s

18.311 Principles of Applied Mathematics
Spring 2014