Shock structure due to higher order effects

Diffusion effects in traffic flow: "look ahead" by drivers.

When the gradients are not small, one must consider the fact that reasonable people drive "preventively" --- i.e.: they attempt to predict "future" traffic conditions, and proceed accordingly. A rough model for this is:
If density is larger ahead, $\rho_x > 0$, slow down below what local density indicates, and conversely. A simple mathematical model incorporating this is:
$q = Q(\rho) - \nu^*\rho_x$ in the conservation of cars law, where $\nu > 0$ is a constant (ν is a diffusivity). This leads to: $\rho_t + (Q(\rho))_x = \nu^*\rho_{xx}$

Here "ν" is small away from shocks. What this means is that, if
L = typical length scale for the traffic flow away from shocks, and
T = typical time scale for the traffic flow away from shocks, then
ν^*T/L^2 is small.

Of course, for the time and space scales near shocks this fails, as we will see next.

The role diffusion plays in stopping steepening and wave breaking. Note that new term adds a contribution to the car flux which is of the same type as the one diffusion (as in heat) causes. As wave steepens, this term gets larger and larger, till it eventually can stop the steepening.
Shock as thin layer where diffusion and nonlinearity balance.
Shock structure argument: the shock zone in space time is very thin, thus shock look ``locally'' like a plane wave. That is: a traveling wave.

Traveling wave for augmented equation.
Substitute $\rho = R((x-s^*t)/\nu)$ into the equation. Then the equation reduces to ode for R.
Study this ode and:

- Analyze conditions under which a wave connecting two given states as $(x-s^*t)/\nu$ to $\pm\infty$ exists. This gives shock as ν vanishes.
- Recover jump and entropy conditions.

ODE is: $R' = Q(R) - [s^*R + \text{const.}]$.
Use graphical analysis: curve Q(R) vs straight line s*R+const.