18.175: Lecture 20

Infinite divisibility and Lévy processes

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Outline

Infinite divisibility

Higher dimensional CFs and CLTs

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Infinitely divisible laws

- Say a random variable X is **infinitely divisible**, for each n, there is a random variable Y such that X has the same law as the sum of n i.i.d. copies of Y.
- What random variables are infinitely divisible?
- Poisson, Cauchy, normal, stable, etc.
- Let's look at the characteristic functions of these objects. What about compound Poisson random variables (linear combinations of independent Poisson random variables)? What are their characteristic functions like?
- What if have a random variable X and then we choose a Poisson random variable N and add up N independent copies of X.
- More general constructions are possible via Lévy Khintchine representation.

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Higher dimensional limit theorems

- Much of the CLT story generalizes to higher dimensional random variables.
- For example, given a random vector (X, Y, Z), we can define $\phi(a, b, c) = Ee^{i(aX+bY+cZ)}$.
- This is just a higher dimensional Fourier transform of the density function.
- The inversion theorems and continuity theorems that apply here are essentially the same as in the one-dimensional case.

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