18.100C Lecture 2 Summary

Axioms of a field.

Theorem 2.1. *In any field, $x \cdot 0 = 0$ for all x.*

Examples. Field with two elements.
Axioms of an ordered field.

Theorem 2.2. *In any ordered field, $1 > 0$.*

Theorem 2.3. *In any ordered field, $x > 0$ if and only if $-x < 0$.*

Corollary 2.4. *In any ordered field, $x^2 \geq 0$, with equality if and only if $x = 0$.*

Least upper bounds. Axiom of the least upper bound. The real numbers.

Theorem 2.5. *There is a unique real number $x > 0$ such that $x^2 = 2$.*

Similarly, existence of square roots for all positive real numbers:

Corollary 2.6. *A real number is nonnegative if and only if it is a square.*

Theorem 2.7. *(Archimedean principle) For every real number x there is a natural number n such that $n > x$.*

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