18.100**C**
Fall 2012

Practice Midterm 2

No notes, textbooks, calculators, or other materials may be used. Please switch off all mobile phones or other electronic devices.

Unless the problem specifically states otherwise, the rules are as follows: you can use any theorem proved in the 18.100C lectures, and any theorem which is in the body of the textbook. However, you should state the theorem clearly when you use it. You may not use any theorems proved in the homework, or ones which are in the problem part of the textbook (if you want to, you have to reproduce their proofs).

1. True or false (proof of counterexample): if $\sum_k a_k$ converges, then so does $\sum_k 2^{-k} a_k$.
2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function which is periodic, meaning that there is some $C > 0$ such that $f(x + C) = f(x)$ for all x. Prove that $f(\mathbb{R})$ is a closed and bounded interval.
3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is uniformly continuous. Prove that f can have "at most linear growth", which means the following. There are constants C, D such that for all $x \in \mathbb{R}$, $|f(x)| < C + D|x|$.
4. Let f be a function which is twice differentiable at p, and such that $f'(p) = 0$, $f''(p) > 0$. Prove that there is a $\delta > 0$ such that if $0 < |x - p| < \delta$, then $f(x) > f(p)$.

Score: 5+5+5+5 = 20 points. A score of 12 or higher is considered a passing grade (this has no concrete consequences, it's just for your information.)