Corrections and Changes to the Third through the Seventh Printings

Revised Oct. 8, 2011

The third printing has 10 9 8 7 6 5 4 3 on the first left-hand page. Later printings end with higher numbers (currently: 4, 5, 6, or 7).

The list below omits:
minor English typos (doubled periods, wrong punctuation, accidental misspellings);
minor non-confusing mathematical typos: poor spacing is the most common.

Bullets mark the more significant changes or corrections: missing or altered hypotheses, non-evident typos, new hints or simplifications, etc.

Double bullets mark new exercises or substantially changed ones, or significant changes to or errors in the text material.

p. 10, Def. 1.6B: *read:* Any such $C \ldots$
• p. 30, Ex. 2.1/3: *replace:* change the hypothesis on $\{b_n\}$ *by:* strengthen the hypotheses (cf. p. 405, Example A.1E for the meaning iof "stronger")
• p. 30, Ex. 2.2/1b: *read:* (make the upper bound sharp)
p. 47, Ex. 3.3/1d: *replace semicolons by commas*

•• p. 48 *Add:*
3-5 Given any c in $\mathbf{R}$, prove there is a strictly increasing sequence $\{a_n\}$ and a strictly decreasing sequence $\{b_n\}$, both of which converge to c, and such that all the a_n and b_n are
(i) rational numbers; (ii) irrational numbers. (Theorem 2.5 is helpful.)

p. 55, line 7: *read:* if $0 < |e_n| < .9$,
p. 57, display (17): *read:* if $0 < |e_n| \leq .2$
• p. 58, Ex. 4.3/2: *Omit.* (too hard)
p. 60, Ans. 4.3/2: *read:* 1024
p. 63, display (9): *delete:* > 0
p. 63, line 11 from bottom: *read:* 5.1/4
p. 68, line 10: *replace:* hypotheses *by:* symbols; *replace* or *by* and
• p. 69, line 9: *read:* strictly increasing, clearly $n_1 \geq 1, n_2 \geq 2$, and so on, so eventually
lines 11, 13: *replace:* $i \gg 1$ *by* $i > N$
p. 73, line 2: *read:* $a_n - L$
p. 73, line 6-: *read:* and estimate it: use 2.4(4), and (16a), suitably applied to $\{b_n\}$.

•• p. 74, Ex. 5.4/1 *Add two preliminary warm-up exercises:*
a) Prove the theorem if $k = 2$, and the two subsequences are the sequence of odd terms a_{2i+1}, and the sequence of even terms a_{2i}.
b) Prove it in general if $k = 2$.
c) Prove it for any $k \geq 2$.

• p. 75, Prob. 5-1(a): *replace the first line of the "proof" by:*
Let $\sqrt{a_n} \to M$. Then by the Product Theorem for limits, $a_n \to M^2$, so that
p. 82, Proof (line 2): *change:* a_n *to* x_n
p. 89, Ex. 6.1/1a: *change* c_n *to* a_n
p. 89, Ex. 6.1/1b *add:* to the limit L given in the Nested Intervals Theorem.

•• p. 89, Ex. 6.2 *add:* **3.** Find the cluster points of the sequence $\{\nu(n)\}$ of Problem 5-4.
•• p. 90, Ex. 6.3 *add:* **2.** Prove the Bolzano-Weierstrass Theorem without using the

Cluster Point Theorem (show you can pick an x_{n_i} in $[a_i, b_i]$).

p. 90, Ex. 6.5/4: *read:* non-empty bounded subsets

p. 95, Display (6): *delete:* e

p. 104, l. 10- *read:* $N + 1$

p. 106, l. 10 *read:* $\sum(-1)^{n+1}/n$

p. 107, l. 2,3 *insert:* this follows by Exercise 6.1/1b, or reasoning directly, the picture

p. 108, bottom half through top p.109 *replace everywhere:* "positive" and "negative" by "non-negative" and "non-positive" respectively

p. 114, line 3- *replace:* $\leq$ *by* $<$

p. 115, line 12- *read:* $|a_n| < 1$

•• Question 8.2/2 *the series is not Abel-summable; replace by:* Show the Abel sum of $0 + 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is the same as its ordinary sum (cf. 4.2).

p. 121, line 9 *read:* 8.4A

•• p. 122, line 12 *replace by*: $d_n - e_n$, where d_n and e_n are respectively the two positive series in the line above.

Replace to the end c_n^+ *and* c_n^- *by* d_n *and* e_n*; add after the next paragraph:*

Since d_n and e_n are positive series, they are absolutely convergent, and

$$\sum |c_n| = \sum |d_n - e_n| \leq \sum(|d_n| + |e_n|) = \sum d_n + \sum e_n,$$

which shows that $\sum c_n$ is also absolutely convergent.

•• p. 124, Problems *add:* **8-2** The multiplication theorem for series requires that the two series be absolutely convergent; if this condition is not met, their product may be divergent.

Show that the series $\displaystyle\sum_0^\infty \frac{(-1)^i}{\sqrt{i+1}}$ gives an example: it is conditionally convergent, but its product with itself is divergent. (Estimate the size of the odd terms c_{2n+1} in the product.)

•• p. 124, 8.2 **2.** $0 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots = \ln(1+x)$; Abel sum is $\ln 2$ (cf. 4.2).

p. 130, line 13: *Fourier analysis* is devoted to studying to what extent periodic functions

• p. 135, *Add hypotheses:* $a_1 > 0$, $f(a_1) = a_1$, $f(a_2) = a_2$.

• p. 143, Example 10.3A and Solution. *in* $x^4 < x^2$, $x^3 < x^2$ *replace* $<$ *by* $\leq$

p. 144, line 4- *read:* non-zero polynomial

• p. 148, Ex. 10.1/7a(ii) *read:* is strictly decreasing

p. 154, first line below pictures: *read:* points of discontinuity

p. 154, line 8 from bottom *insert paragraph:*

On the other hand, functions like the one in Exercise 11.5/4 which are discontinuous (i.e., not continuous) at every point of some interval are somewhat pathological and not generally useful in applications; in this book we won't refer to their x-values as points of discontinuity since "when everyone is somebody, then no one's anybody". If necessary, we will use the oxymoronic "non-isolated point of discontinuity".

p. 156, line 4: *read:* In (8) below, the first limit exists if and only if the second and third exist and are equal;

p. 157, line 5: *read:* $x \ll -1$

p. 161, line 11: *delete ;* , line 12: *read* $<$, line 13 *read* $\leq$

• p. 164, *read:* **Thm.11.4D′** *Let $x = g(t)$, I be a t-interval, J be an x-interval. Then $g(t)$ continuous on I, $g(I) \subseteq J$, and $f(x)$ continuous on $J \Rightarrow f(g(t))$ continuous on I.*

p. 167, Ex. 11.1/4 *read:* exponential law, $e^{a+b} = e^a e^b$,

• p. 168, Ex. 11.3/3 *read:* b) $\lim_{x \to 0^-} \int_0^1 t^2/(1 + t^4 x)\, dt = 1/3$.

p. 168, Ex. 11.3/5 *add:* As $x \to x_0$,

- p. 168, Ex. 11.5/2: *rewrite:* Prove $\lim\limits_{x\to\infty} \sin x$ does not exist by using Theorem 11.5A.

p. 180, Ex. 12.1/3: *read:* a polynomial

p. 180. Ex. 12.2/1: *add at end:* Make reasonable assumptions.

p. 181, Ex. 12.2/3: *change:* solutions *to* zeros

p. 183, 12.1/4: *read:* $\log_2[(b-a)/e]$

- p. 192, Ex. 13.1/2 *renumber as 13.2/2, and change part (b) to:*

  13.2/2b Is there a continuous function which satisfies the conditions of part (a)? Justify your answer.

p. 193, Ex. 13.5/2 *change the two R to* $\mathbf{R}$

p. 194, Problem 13-7 last two lines, *read:* but for the part of that argument using the compactness of $[a,b]$, subnstitute part (a) of 13-6 above.)

p. 203, Theorem 14.3B: *label:* Local Extremum Theorem

p. 204, line 4: *read:* an open I

p. 221, line 11- *read:* (a,b)

Sol'n 15.4/1c: *read:* not one-third!

p. 227, line 2: *read:* $f'(x)$ not convex

p. 228, Ex. 16.1/1a,b *read:* $(0,1]$; Ex. 1b: $x - x^2/2$

p. 228, Ex. 16.2/1 *replace:* the second derivative test *by* each statement in (8)

p. 230, Ans. 16.1/2 *change 9 to 0*

p. 231, line 3- *change k to a*

p. 235, display (15): *change* $0 < |c| < |x|$ *to* $\begin{cases} 0 < c < x, \\ x < c < 0\ . \end{cases}$; *delete next two lines*

- p. 243, Example 18.2, Solution, lines 4 and 7 *read:* $[0, x_1]$

p. 245 lines 1,2: $f(x_{i-1}))$, line 15: two underscripts: $[\Delta x_i]$

- p. 248, Ex. 18.2/1 *add:* Hint: cf. Question 18.2/4; use $x_i^2 - x_{i-1}^2 = (x_i + x_{i-1})(x_i - x_{i-1})$; i.e., do it directly, not using the general theorems in 18.3.

p. 248, Ex. 18.3/1 *replace n by k everywhere*

•• p. 260, Defn. 19.6 *read:* $a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b$

add at end: and has finite left and right limits at each x_i (just a finite one-sided limit at x_0, x_n). (Thus $f(x)$ can have discontinuities only at the x_i, and they are jump or removable discontinuities.)

- p. 261, Solution. a) $\tan x$ is piecewise monotone with respect to $< 0, \pi/2, 3\pi/2, 2\pi >$, but not piecewise continuous since its limits at $\pi/2$ and $3\pi/2$ are not finite.

(b) *read:* $[1/(n+1)\pi, 1/n\pi]$

p. 261, Lemma 19.6 *rename: Endpoint Lemma*

p. 261, line 7- *replace:* $[c,d]$ *by* $[a,b]$

p. 265, Ex. 19.6/1b line 2 *replace:* $f(x)$ *by* $p(x)$

p. 273, line 2- : *read:* (cf. p. 271)

p. 282, line 2- : *read:* by interpreting the integral and limit geometrically

p. 289, Ans. 20.5/1: *read:* 1024

p. 291, Ex. 21.1B - line 3: *read:* $\lim_{R\to\infty} \int_{-R}^{0}$

line -2: *read:* for $p > 1$,

p. 307, Example 22.1C *read:* Show: as $n \to \infty$, $\dfrac{n}{1+nx} \ldots$

p. 310, Theorem 22.B *read:* $\sum_0^\infty M_k$

p. 316, Theorem 22.5A: *delete:* for all $n \geq 0$

p. 322, Ex. 22.1/3 *read:* $u_k(x) =$

p. 332, middle *delete both* $\aleph_1$, *replace the third display by:* $\aleph_0 = N(\mathbf{Z}) < N(S) < N(\mathbf{R})$

p. 335, lines 6-,7-: *read:* bounded and have only a finite number of jump discontinuities

•• p. 340, *delete: last 9 lines of text before Questions 23.4*

p. 350, line 10-: *read:* Subsequence Theorem 5.4

p. 351, line 5-: *read:* infinite quarter-planes containing the x-axis and lying between ...

• p. 353, line 4-: *read:* 24.4A;

line 2-: *read:* $x + y = 2$

p. 354, Theorem 24.5B: *read:* for all $\mathbf{x}_n$

line 7- *read:* $f(\mathbf{x}_n)$

p. 357, Theorem 24.7B, line 2 *read:* non-empty compact set S;

line 6 *read:* bounded and non-empty;

p. 367, line 15- *add:* Or make up a simple direct proof.

p. 369, Theorem 25.3A: (i) *read:* then $S =$; (ii) *read:* $S = \bigcup_{i \epsilon I} U_i$

p. 377, Ex. 26.2B, Solution line 2: *read:* $(-\infty, \infty)$ *change* $(*)$ *to* (5) *throughout*

p. 385, line 2- *read:* $\int_0^1$

•• p. 388, footnote *replace by:* We prove the first inequality in (7), which is the analog – for absolutely convergent improper integrals – of the infinite triangle inequality for sums.

For a fixed x, we have by the Absolute Value Theorem for integrals (19.4C)

$$\left|\int_R^S f(x,t)\,dt\right| \le \int_R^S \left|f(x,t)\right| dt, \quad \text{for all } S > R,\ R \text{ fixed.}$$

As $S \to \infty$, the right side has the limit $\int_R^\infty |f(x,t)|\,dt$, since the integral $\int_R^\infty f(x,t)\,dt$ is assumed to be absolutely convergent.

The left side has the limit $|\int_R^\infty f(x,t)\,dt|$, since the integral is convergent (by theorem 21.4), and $|\ |$ is a continuous function.

Finally, by the Limit Location Theorem 11.3C (21), the inequality is preserved as $S \to \infty$.

p. 399, line 18- *read:* $a(b + c) = ab + ac$

p. 404, Example A.1C(i): *read:* $a^2 + b^2 = c^2$

p. 415, Ex. A.4/6 *read:* Fermat's Little Theorem is the basis of the RSA encryption algorithm, widely used to guarantee website security.

p. 417, A.4/1 line 1: *read:* both sides are 1

A.4/2 line 1: *read:* $2^n + 1$

•• p. 429 last 5 lines: *replace sentences by:* As the picture shows, since $|f'(x)| > 1.2$ on $[.7, 1]$, we will have its reciprocal $|g'(x)| < 1/1.2 \approx .8$ on the interval $[0, f(.7)] = [0, .83]$.

This shows Pic-2 is satisfied for $g(x)$ on the interval $[0, .83]$; the picture shows the root of $x = g(x)$ will lie in this interval. Thus the Picard method is applicable to $x = g(x)$. Starting with say .7, it leads to a root $\approx .76$.

p. 436, Remarks, first paragraph: *replace* x^3 *by* x^4

p. 439, top half: *change* p *and* q *to* P *and* Q (to avoid confusion with the use of the real number p in Example D.4)

p. 442, line 2: *read:* $\ge$ line 6: *read:* $\le$

•• p.443, Ex. D.2/4: *read:* Find, by calculating the derivatives for $x \ne 0$ and using undetermined coefficients, a second-order linear homogeneous D.E. satisfied by

$$y = x^4 \sin(1/x),\ y(0) = 0, \ldots$$

p. 459, ruler function: *read:* 169

18.100A Introduction to Analysis
Fall 2012