

## 18.100A Fall 2012: Assignment 20

*The rules are the same as for previous assignments.*

**Reading: 22.5, 22.6** (through p. 319)

Differentiation of sequences and series; application to power series.

The end result of applying the theorems of this chapter to power series is this theorem, whose three parts can be found, with more detail, in sections 22.3, 22.4, and 22.6. You can use it in citations.

Power series theorems which require uniform convergence in their hypotheses or conclusions should be cited separately, by section number.

**Power Series Theorem.** A power series  $\sum a_n x^n = f(x)$  with radius of convergence  $R$  represents a function  $f(x)$  defined, continuous, integrable, and differentiable for  $|x| < R$ , whose integral  $\int_a^x f(t) dt$  and derivative  $f'(x)$  can be calculated by integrating and differentiating the series term-by-term.

Note that though the series may also converge for  $x = R$  or  $x = -R$ , the function  $f(x)$  need not be continuous or differentiable at these points, and the series may not converge uniformly over the interval  $(-R, R)$ .

(It's customary to do term-by-term integration of power series by finding the antiderivative of each term, lumping all the arbitrary constants from the antiderivatives together into a single constant  $C$ , and evaluating  $C$  by substituting a numerical value for  $x$  into the series.)

**Problem 1.** (3: 1.5,1.5) a) Work 22.5/1      b) Work 22.5/2

Since these are not power series, for both be careful to cite the theorem which justifies your statements, and to verify that the hypotheses of those theorems hold for the functions and series in these two exercises.

**Problem 2.** (2) Work 22.6/1

Use a dependent variable  $y = f(x)$ , and show using term-by-term differentiation that  $y$  satisfies the first-order differential equation  $y' - xy = 1$ .

Solve the equation a different way by multiplying both sides by the integrating factor  $e^{-x^2/2}$ , then integrating both sides from 0 to  $x$  and using the initial condition. In this way determine the sum  $f(x)$  of the series explicitly. (It involves a non-elementary integral.) Over what interval is the solution valid?

**Problem 3.** (2) Work 2.6/5ab, using term-by-term integration. Don't forget the arbitrary constant  $C$ . (Part (b) uses part (a).)

**Problem 4.** (3: 1,2)

a) Derive the series for  $\ln(1+x)$  by using differentiation, then term-by-term integration.

b) Work Problem 22-1.

Let  $f(x)$  be the sum of the series. You will need the result in Part (a). Cite the theorems used; verify hypotheses. Include a suitable definition for  $f'(0)$ , with justification.

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