Series & Convergence

$S = \sum_{n=0}^{\infty} A_n$: infinite series. When does $\sum_n$ converge to something finite?

- Ratio Test: Suppose $L = \lim_{n\to\infty} \left| \frac{A_{n+1}}{A_n} \right|$: exists

  if $L < 1 \to$ series converges absolutely $\left(\sum |A_n| < \infty\right)$

  if $L > 1 \to$ series diverges

  $L = 1 \to$ inconclusive

ex $\sum_{n=0}^{\infty} \underbrace{a_n (z-z_0)^n}_{A_n} = S(z)$

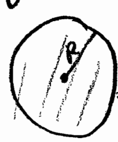
$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}\,(z-z_0)^{n+1}}{a_n\,(z-z_0)^n} \right| = |z-z_0| \boxed{\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|} \leftarrow \bar{L}$$

$L < 1 \to$ series converges absolutely

$\hookrightarrow |z-z_0| < \frac{1}{\bar{L}} = R$

$L > 1 \to$ series diverges

$\hookrightarrow |z-z_0| > \frac{1}{\bar{L}} = R$



$\leftarrow S$ converges inside

- Root Test: Suppose $L = \lim_{n\to\infty} \sqrt[n]{|A_n|}$: exists

  if $L < 1 \to$ series converges absolutely

  if $L > 1 \to$ series diverges