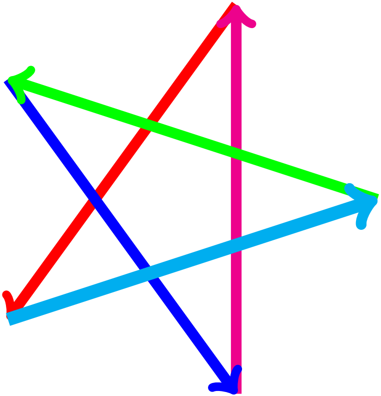
Studio 2

18.05 Spring 2014

Jeremy Orloff and Jonathan Bloom



Expected Value

If X is a random variable the takes values $x_1, x_2, \ldots, x_n$ then the *expected value* of X is defined by

$$E(X) = p(x_1)x_1 + p(x_2)x_2 + \ldots + p(x_n)x_n = \sum_{i=1}^{n} p(x_i)\, x_i$$

- Weighted average
- Measure of central tendency

Properties of $E(X)$

1. $E(X + Y) = E(X) + E(Y)$
2. $E(aX + b) = aE(X) + b$
3. $E(h(X)) = \sum_{i} h(x_i)\, p(x_i)$

Examples

Example 1. Find $E(X)$

1.	X:	3	4	5	6
2.	pmf:	1/4	1/2	1/8	1/8

3. $E(X) = 3/4 + 4/2 + 5/8 + 6/8 = 33/8$

Example 2. Suppose $X \sim$ Bernoulli(p). Find $E(X)$.

1.	X:	0	1
2.	pmf:	$1-p$	p

3. $E(X) = (1-p)\cdot 0 + p \cdot 1 \; = \; p.$

Example 3. Suppose $X \sim$ Binomial$(12, .25)$. Find $E(X)$.
$X = X_1 + X_2 + \ldots + X_{12}$, where $X_i \sim$ Bernoulli$(.25)$. Therefore

$$E(X) = E(X_1) + E(X_2) + \ldots E(X_{12}) = 12 \cdot (.25) = 3$$

In general if $X \sim$ Binomial(n, p) then $E(X) = np$.

Board Question

Suppose (hypothetically!) that everyone at your table gets up, does a board question, and sits back down at random (i.e., all seating arrangements are equally likely).

What is the expected number of people who return to their original seat?

R Exercises

Suppose $Y \sim$ Binomial(8,.6).

1. Run a simulation with 1000 trials to estimate $P(Y = 6)$ and $P(Y <= 6)$

2. Use R and the formula for binomial probabilities to compute P(Y=6) exactly.

R Exercises

3. A friend has a coin with probability .6 of heads. She proposes the following gambling game.

- You will toss it 10 times and count the number of heads.
- The amount you win or lose on k heads is given by $k^2 - 7k$

(a) Plot the payoff function.

(b) Make an exact computation using R to decide if this is a good bet.

(c) Run a simulation and see that it approximates your computation in part (b).