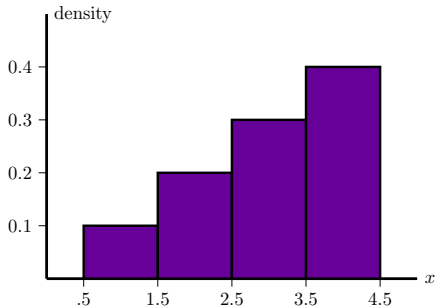
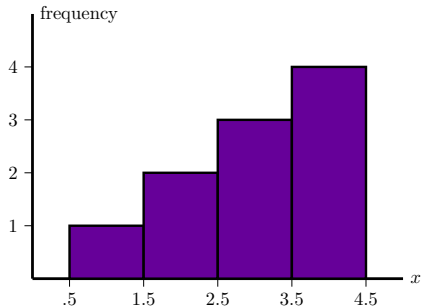


# Studio 3

## 18.05 Spring 2014

Jeremy Orloff and Jonathan Bloom



## Concept questions

Suppose  $X$  is a continuous random variable.

a) What is  $P(a \leq X \leq a)$ ?

b) What is  $P(X = 0)$ ?

c) Does  $P(X = 2) = 0$  mean  $X$  never equals 2?

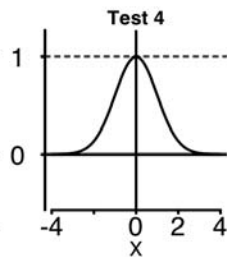
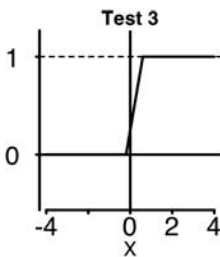
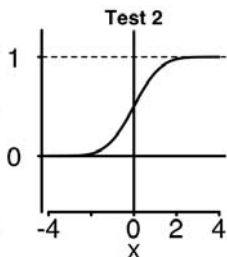
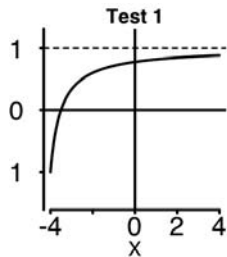
**answer:** a) 0

b) 0

c) No. For a continuous distribution any single value has probability 0. Only a range of values has non-zero probability.

## Concept question

Which of the following are graphs of valid cumulative distribution functions?



Add the numbers of the valid cdf's and click that number.

**answer:** Test 2 and Test 3.

## Solution

Test 1 is not a cdf: it takes negative values, but probabilities are positive.

Test 2 is a cdf: it increases from 0 to 1.

Test 3 is a cdf: it increases from 0 to 1.

Test 4 is not a cdf: it decreases. A cdf must be non-decreasing since it represents *accumulated* probability.

## Exponential Random Variables

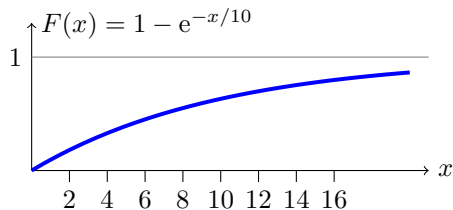
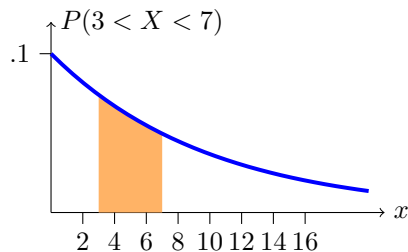
Parameter:  $\lambda$  (called the rate parameter).

Range:  $[0, \infty)$ .

Notation:  $\text{exponential}(\lambda)$  or  $\text{exp}(\lambda)$ .

Density:  $f(x) = \lambda e^{-\lambda x}$  for  $0 \leq x$ .

Models: Waiting time



Continuous analogue of geometric distribution –memoryless!

## Uniform and Normal Random Variables

**Uniform:**  $U(a, b)$  or  $\text{uniform}(a, b)$

Range:  $[a, b]$

$$\text{PDF: } f(x) = \frac{1}{b - a}$$

**Normal:**  $N(\mu, \sigma^2)$

Range:  $(-\infty, \infty]$

$$\text{PDF: } f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

<http://web.mit.edu/jorloff/www/18.05/applets/probDistrib.html>

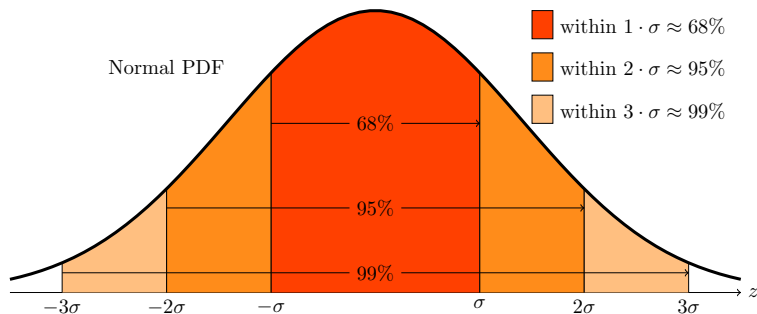
## Table questions

Open the applet

<http://web.mit.edu/jorloff/www/18.05/applets/probDistrib.html>

1. For the **standard normal** distribution  $N(0, 1)$  how much probability is within 1 of the mean? Within 2? Within 3?
2. For  $N(0, 3^2)$  how much probability is within  $\sigma$  of the mean? Within  $2\sigma$ ? Within  $3\sigma$ .
3. Does changing  $\mu$  change your answer to problem 2?

# Normal probabilities



Rules of thumb:

$$P(-1 \leq Z \leq 1) \approx .68,$$

$$P(-2 \leq Z \leq 2) \approx .95,$$

$$P(-3 \leq Z \leq 3) \approx .997$$



## Download R script

Download studio3.zip and unzip it into your 18.05 working directory.

Open studio3.r in RStudio.

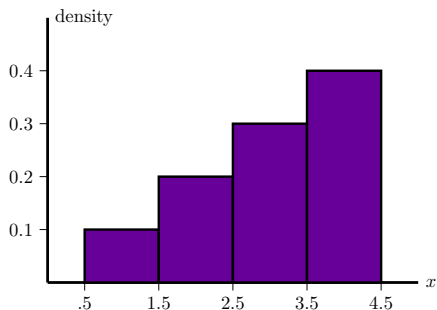
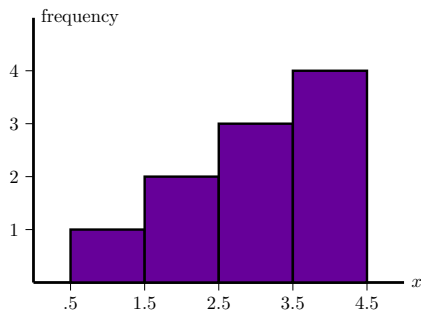
# Histograms

Will discuss in more detail in class 6.

Made by 'binning' data.

**Frequency:** height of bar over bin = # of data points in bin.

**Density:** area of bar over bin is proportional to # of data points in bin. Total area of a density histogram is 1.



## Histograms of averages of $\exp(1)$

1. Generate a frequency histogram of 1000 samples from an  $\exp(1)$  random variable.
2. Generate a density histogram for the average of 2 independent  $\exp(1)$  random variable.
3. Using `rexp()`, `matrix()` and `colMeans()` generate a density histogram for the average of 50 independent  $\exp(1)$  random variables. Make 10000 sample averages and use a binwidth of .1 for this. Look at the spread of the histogram.
4. Superimpose a graph of the pdf of  $N(1, 1/50)$  on your plot in problem 3. (Remember the second parameter in  $N$  is  $\sigma^2$ .)

Code for the solutions is at

<http://web.mit.edu/jorloff/www/18.05/r-code/studio3-sol.r>

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## 18.05 Introduction to Probability and Statistics

Spring 2014

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