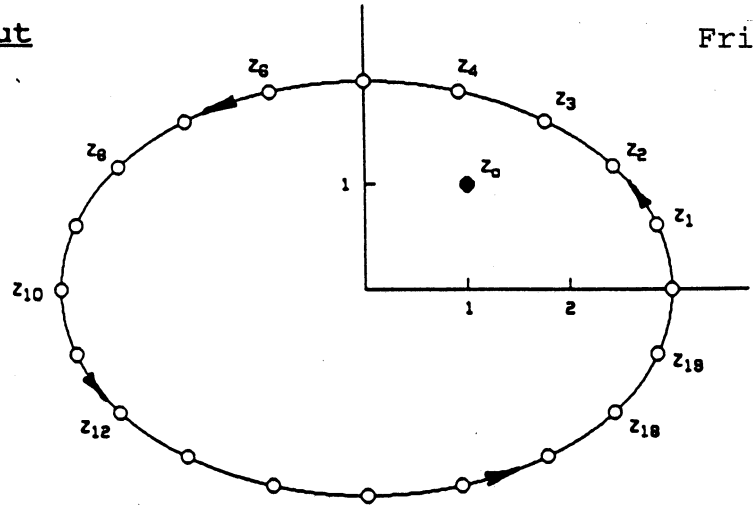

Unlike humans, computers never — well, hardly ever — evaluate messy integrals by "doing" them exactly. Instead, they handle such tasks mostly by converting those into finer and finer **sums** of N separately-evaluated pieces ... and of course their supposedly smart homo sapiens bosses are strongly urged to check and double-check soon afterwards that the sums so cranked out using a variety of **different** N's indeed look as if they have converged adequately to agree at least with each other, and thus perhaps even with the true values of the integrals!

Here are some such numerical results just obtained for the integrals

$$I_m = \frac{1}{2\pi i} \oint \frac{z^3 + 5z + 2}{(z - z_o)^{m+1}}\, dz \quad , \quad \text{for } m=0,1,2,3,4 \; ,$$

each taken counterclockwise once around the ellipse $x^2/9 + y^2/4 = 1$ in the complex $z = x + iy$ plane, and again using $z_o = 1 + i$ for the sake of this illustration:

	m = 0		m = 1		m = 2		m = 3		m = 4	
N	Re	Im	Re	Im	Re	Im	Re	Im	Re	Im
10	5.310101	6.977182	5.397032	4.597659	0.347379	1.008208	-3.116812	2.600923	1.369181	4.990581
15	5.049101	6.970114	4.910377	5.622131	1.731604	2.864570	-.222895	2.411595	2.731509	3.097200
20	5.005491	6.990684	4.934419	5.929650	2.621742	3.178606	1.129170	1.176467	2.370213	0.594516
25	5.000075	6.997964	4.978551	5.993216	2.931837	3.101173	1.269728	0.338175	1.058773	-.385760
30	4.999830	6.999656	4.995042	6.001022	2.997789	3.032813	1.131548	0.048367	0.267488	-.349917
35	4.999941	6.999958	4.999159	6.000722	3.003655	3.007557	1.041112	-.007459	0.015960	-.152891
40	4.999986	6.999998	4.999917	6.000223	3.001690	3.001175	1.008944	-.007573	-.020661	-.044881
45	4.999998	7.000001	5.000007	6.000050	3.000487	3.000048	1.001079	-.002946	-.012200	-.008576
50	5.000000	7.000000	5.000006	6.000008	3.000104	2.999855	0.999869	-.000799	-.004265	-.000394
55	5.000000	7.000000	5.000002	6.000001	3.000016	2.999980	0.999874	-.000159	-.001077	0.000486
60	5.000000	7.000000	5.000000	6.000000	3.000001	2.999994	0.999954	-.000019	-.000187	0.000254
65	5.000000	7.000000	5.000000	6.000000	3.000000	2.999999	0.999988	0.000001	-.000010	0.000081
70	5.000000	7.000000	5.000000	6.000000	3.000000	3.000000	0.999998	0.000002	0.000008	0.000019
75	5.000000	7.000000	5.000000	6.000000	3.000000	3.000000	1.000000	0.000001	0.000004	0.000003
80	5.000000	7.000000	5.000000	6.000000	3.000000	3.000000	1.000000	0.000000	0.000001	0.000000
85	5.000000	7.000000	5.000000	6.000000	3.000000	3.000000	1.000000	0.000000	0.000000	0.000000
90	5.000000	7.000000	5.000000	6.000000	3.000000	3.000000	1.000000	0.000000	0.000000	0.000000
95	5.000000	7.000000	5.000000	6.000000	3.000000	3.000000	1.000000	0.000000	0.000000	0.000000
100	5.000000	7.000000	5.000000	6.000000	3.000000	3.000000	1.000000	0.000000	0.000000	0.000000

Estimating exp(1+i) via the means of N measurements

of exp(z) made at equally-spaced locations around a unit circle centered on z = 1 + i:

N	phase	Real	Imag
3	0.0	1.715 520 166 324	2.671 764 358 834
4	0.0	1.529 925 949 753	2.382 718 492 295
5	0.0	1.480 933 460 816	2.306 417 211 574
6	0.0	1.470 733 795 676	2.290 532 174 297
7	0.0	1.468 985 347 460	2.287 809 127 540
8	0.0	1.468 730 365 857	2.287 412 017 221
9	0.0	1.468 697 987 243	2.287 361 590 517
10	0.0	1.468 694 344 649	2.287 355 917 513
11	0.0	1.468 693 976 710	2.287 355 344 482
12	0.0	1.468 693 942 982	2.287 355 291 954
13	0.0	1.468 693 940 152	2.287 355 287 546
14	0.0	1.468 693 939 933	2.287 355 287 205
15	0.0	1.468 693 939 917	2.287 355 287 181
16	0.0	**1.468 693 939 916**	**2.287 355 287 179**

N	phase	Real	Imag
3	0.5	1.225 947 425 029	1.909 299 989 761
4	0.5	1.407 534 781 961	2.192 105 542 146
5	0.5	1.456 455 228 481	2.268 294 623 451
6	0.5	1.466 654 090 288	2.284 178 409 611
7	0.5	1.468 402 532 405	2.286 901 446 870
8	0.5	1.468 657 513 975	2.287 298 557 137
9	0.5	1.468 689 892 589	2.287 348 983 841
10	0.5	1.468 693 535 183	2.287 354 656 845
11	0.5	1.468 693 903 122	2.287 355 229 876
12	0.5	1.468 693 936 850	2.287 355 282 404
13	0.5	1.468 693 939 680	2.287 355 286 812
14	0.5	1.468 693 939 899	2.287 355 287 153
15	0.5	1.468 693 939 915	2.287 355 287 177
16	0.5	**1.468 693 939 916**	**2.287 355 287 179**

PS: Any set of points with phase = 0 includes z = 2 + i whereas one with phase = 0.5 exactly straddles it.