18.03SC Differential Equations, Fall 2011
Transcript – Laplace Transform: Basics

PROFESSOR: Welcome back. So in this session we're going to look at Laplace transform. And we'll start with asking you to recall the definition that you saw in class, then to use the definition to compute the Laplace transform of the function 1, exponential At, and the delta function. For each one of these, give the domain of definition or convergence of the od integral. For the last question, you're asked to use the results of question 2 to give the Laplace transform of this linear combination of functions. In the last part, you're asked to compute Laplace transform, cosine and sine. So why don't you pause the video, take a few minutes, and work through that.

Welcome back. So let's start with the definition. Laplace transform of the function s was defined as the integral from 0 minus to infinity of the function f of t exponential minus st dt. So note here, the interval of integration is 0 minus to infinity.

So using this definition, we can go ahead and compute our first Laplace transform, L of 1. So I'm just going to substitute 1 in that integral, which gives me basically exponential minus st dt, which is just the integral of the exponential minus st over minus s from 0 minus to infinity. And if I expend this, basically, I end up with 1/s. The minus reverses the order of integration, so I start with 0, which is 1, minus the limit when t goes to infinity, my exponential minus st.

So here the sign of s becomes important. If s was positive, then this term would go to 0 as t goes to infinity. If s is negative, then this term diverges, and so we're not anymore in the domain of convergence of the Laplace integral.

But really, s could be also complex. So what we're interested in is really the sign of the real part of s. So if the real part of s is positive, this term is goes to 0, and the Laplace transform of 1 is just 1/s. And if the real part of s is negative, then the Laplace diverges. So the domain of convergence in which you want to be on is the one where the real part of s is positive.

OK. So let's move to the second one. The second one is a Laplace of exponential at. So we can move a bit faster now, and just merge the two exponentials. Exponential minus 0 to infinity to 0 to infinity of this exponential. Clearly this is just, again, a case of exponential integration between the two bounds.

And here again we're going to hint a problem with the domain of convergence where we need-- so let me just write these again. So we're going to have here a minus-- so we have our a minus s. So we have the limit again when t goes to infinity of exponential minus s plus a capital T minus 1.

And here, again, we need to impose the condition that the real part of a minus s be negative to have the domain of convergence of the integral. And then we're left with the Laplace interval being over 1 over s minus a. If the real part is positive, then we have divergence. So the domain of convergence of this Laplace is the one defined by the real part of a less than the real part of s.

OK. So let's do the last one. The last one is the Laplace transform of the delta function that we saw before. That's 0 minus to infinity delta exponential minus st dt.

So from the previous recitations, we saw that on this domain, from 0 minus to infinity, the delta is 0 everywhere except at 0, where we basically assigned a value of this function at t equal to 0. So basically we're just left with exponential 0, which is 1. And this computation was really easy, due to the properties of the delta function.

So that ends roughly this first part, except that you can also notice here that the domain of convergence for the Laplace for delta is all s. There's no condition.

So the last part, next question, asked us to compute the Laplace transform of a linear combination of functions. So this one is 7 plus 8 exponential 2t plus 9 exponential minus 3t.

So here I've used solely Laplace as an integral, and so the Laplace transform of this linear combination of functions is the linear combination of the Laplace transform of the functions individually. And so we can just rewrite this as 7 Laplace of 1 plus 8 Laplace of exponential 2t plus 9 Laplace exponential minus 3t.

And here we can see how we can recycle the results from the previous part, as we computed the Laplace transform of 1, and we computed the Laplace transform exponential at, which we're going to be able to apply in these two cases. So we can write the results directly here. And I'm just going to not rewrite everything, just include it.

So the Laplace of 1, we found it earlier to be 1/s. The Laplace of exponential 2t we just found here, and would be s minus 2. The Laplace of exponential minus 3t would be s minus minus 3, so it's s plus 3 with the 9. And we're done.

So for the last part, you're asked to compute the Laplace transfer of cosine and sine, and you should know these by heart. But just a trick to remember it-- I just want to remind you, again, of the linearity and the fact that you could also use the formula. Given what we just derived, you could also write this, again due to the linearity of the integral as a linear combination of Laplace of cosine and sine here.

And we know that the Laplace of the exponential at is just what we computed here. So that would be s minus i omega, which you can just rewrite like this. And then identify just the real part with the real part, the imaginary part with the imaginary part, which finishes our problem. And this is a quick way of checking that you have that, right? To give you the Laplace transforms of cosine and sine.

So that ends the problem for today. The key point was just remembering the definition of the Laplace transform, and then learning how to use it for different cases, and identify the domains of convergence.

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