Solutions to Polynomial Input

Quiz: Which of the following are true about the differential equation $3x^{(4)} + 2x^{(3)} + x'' - x' + 4x = 2t^2 + 1$?

Choices:

a) It has no polynomial solutions.

b) It has exactly one polynomial solution.

c) It has many polynomial solutions.

d) All its solutions are polynomials.

e) We can't say from the information given.

Answer: The answer is b.

The method of undetermined coefficients says there will be a particular solution of the form $x_p = At^2 + Bt + C$. Therefore there is at least one polynomial solution.

The general solution is of the form $x = x_p + x_h$, where x_h is a homogeneous solution. Since 0 is not a root of the characteristic equation, every (nonzero) homogeneous solution is a combination of exponentials and/or sinusoidal functions. Therefore x is a polynomial only for the case $x_h = 0$. That is, x_p is the only polynomial solution.

By the way, $x_p = \frac{1}{2}t^2 + \frac{1}{4}t + \frac{1}{16}$.

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