18.03SC Differential Equations, Fall 2011
Transcript – Homogeneous Constant Coefficient Equations: Real Roots

PROFESSOR: Hi, everyone. Welcome back. So today, we're going to take a look at homogeneous equations with constant coefficients, and specifically, the case where we have real roots. And we'll start the problem off by looking at the equation x dot dot plus 8x dot plus 7x equals 0.

And we're asked to find the general solution to this differential equation. And then we also have the question, do all the solutions go to 0 as t goes to infinity? And then for part B, we're going to take a look at just the differential equation y dot equals negative ky. So this is the same equation that we've seen in past recitations.

And we're just going to show that we can use this method to solve this differential equation and obtain the same result. And then lastly, we're asked, or we're told that we have eight routes to an eighth order differential equation, negative 4, negative 3, negative 2, negative 1, 0, 1, 2, and 3. And we're asked, what is the general solution. So why don't you take a moment and try and work these problems out, and I'll be back in a minute.

Hi, everyone. Welcome back. OK, so we're asked to find the general solution to x double dot plus 8x dot plus 7x equals 0. And we see that this is a differential equation, it's linear, and it has constant coefficients. And whenever we have a differential equation that's linear with constant coefficients, one of the standard ways to generate the solution is to seek what sometimes mathematicians call an ansatz, but it's to try a solution of the form x is equal to a constant times e to the st.

And if we substitute a solution n of this form, we see that taking the second derivative of this function pulls down two s's. One derivative pulls down one s. We have no derivatives here. And we also have, on each term, a factor of c times e to the st. And we want this to be 0.

So specifically, ce to the st can't be 0 for all time. So the only way that this can hold is if s squared plus 8s plus 7 equals 0. So what this means is if we choose s to solve this polynomial, then x equals ce to the st will be the solution. And this will be the solution for any constant, c.

OK, so what are the roots to this algebraic equation. Well, we can factorize it. The roots are going to be negative 7 and negative 1. And notice how this whole process has turned a differential equation into a simpler algebraic equation. So if we can solve the algebraic equation, then we can solve the differential equation.

OK, so the general solution. Well, we've just shown that we can take any constant times e to the st, provided s is equal to negative 1 or negative 7. So the general solution is going to be some constant, c1, times e to the minus t, plus c2 can be a different constant, e to the minus 7t.

So notice how there's two constants in the final solution. And the reason there's two constants is because we started out with a second-order differential equation. So, in some sense, for each order of the differential equation, we always have one constant. It's almost as if for each time we integrate, we have a constant of integration. So at the end of the day, we have two constants in our general solution.

As part of part A, we're also asked for any solution to this differential equation, does the solution go to 0 as t goes to infinity? Well, the general solution has this form. So for any constant c1 and c2, the solution is c1 e to the minus t plus c2, e to the minus 7t.

And we see that no matter what c1 and c2 are, this term, as t goes to infinity, is multiplied by e to the minus t, which goes to 0. And the second term also goes to 0. So as t goes to infinity, both e to the minus t and e to the minus 7t both go to 0. So that means that any constant times e to the minus t plus any constant times e to the minus 7t must also go to 0. So hence, x of t goes to 0 as t goes to infinity.

OK. For part B, we have the differential equation y dot equals negative ky. And this is the first-order linear differential equation with constant coefficients. And we're going to use the same trick.

We let y is equal to c times e to the st. And we see that the characteristic equation in this case, it's not a polynomial. It's just s, s is equal to negative k. So we get y is equal to c e to the negative kt is the general solution.

And this is exactly what we had in previous recitations, when we used, for example, integrating factors to solve this very same differential equation. So this just shows that we can use the same method to solve first-order linear differential equations.

OK. Now, lastly, we're given eight roots to an eighth-order differential equation. An eight-order differential equations with constant coefficients. So I'll just write out the roots again. So we're told the roots are negative 4, negative 3, negative 2, negative 1, 0, 1, 2, and 3. And in general, the solution to an eighth-order differential equation whose roots to the characteristic polynomial are negative 4 through 3, the general solution, x of t, is going to be a constant c1 times e to the power of the first root, which will be minus 4t, plus c2 e to the minus 3t.

And of course, we take different constants for each term. c3 e to the minus 2t, plus c4 e to the minus t, plus c5. And now for this term, it should be e to the 0t, but e to the 0t is just 1. So the zero root is just going to give us a constant c5. And we have c6 to the t, plus c7 to the e to the 2t. And then plus c8 e to the 3t.

So the solution has eight terms and eight constants. And just for fun, we can ask, does every solution to this differential equation go to 0 as t goes to infinity. And the answer is no. In fact, although each term with a negative root does go to zero as t goes to infinity, there are three terms that go to positive infinity as t goes to infinity, and there's one term that just stays constant. So in general, as t grows, goes to infinity, these terms will become very large and won't necessarily go to 0. Well, they'll never go to 0.

So I'd just like to conclude there, and I'll see you next time.