Resonant Response Formula

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Exercise. Find the general solution to

$$x'' + 8x' + 7x = 2e^{-t}.$$

Answer.
The characteristic polynomial is

$$p(r) = r^2 + 8r + 7.$$

This has roots -7 and -1. Thus $p(-1) = 0$ and we can't use the exponential response formula. We must use the resonant response formula instead. So we get

$$x_p = \frac{2}{p'(-1)} te^{-t} = \tfrac{1}{3} te^{-t}$$

as a particular solution. The general solution to the associated homogeneous problem is

$$x_h = c_1 e^{-7t} + c_2 e^{-t},$$

and the final solution is

$$x = x_h + x_p = c_1 e^{-7t} + c_2 e^{-t} + \tfrac{1}{3} te^{-t}.$$

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