

Resonant Response Formula

Exercise. Find the general solution to

$$x'' + 8x' + 7x = 2e^{-t}.$$

Answer.

The characteristic polynomial is

$$p(r) = r^2 + 8r + 7.$$

This has roots -7 and -1 . Thus $p(-1) = 0$ and we can't use the exponential response formula. We must use the resonant response formula instead. So we get

$$x_p = \frac{2}{p'(-1)}te^{-t} = \frac{1}{3}te^{-t}$$

as a particular solution. The general solution to the associated homogeneous problem is

$$x_h = c_1e^{-7t} + c_2e^{-t},$$

and the final solution is

$$x = x_h + x_p = c_1e^{-7t} + c_2e^{-t} + \frac{1}{3}te^{-t}.$$

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