18.03SC Differential Equations, Fall 2011
Transcript – Solutions of First Order Linear Equations

PROFESSOR: Welcome back. In this session, we're going to tackle initial value problem, y dot plus ty equals to t. And this initial value problem is going to be subject to the initial condition, y of 0 equals to 3. We are going to use the method of integration factor. And what I want you to use is both definite integrals and indefinite integrals. So why don't you take a few minutes to think about the problem? And we will be right back.

Welcome back. So I hope that you worked out the first part of the problem. So what are we going to do to solve this ODE. First, we need to review what is the method of integrating factor. So when we use the integrating factor, basically, we're trying to write down our ODE in a different form by introducing a function u. And the goal is to find the function u such that we can rewrite this left-hand side as the derivative of the product y dot u.

So in this case, if we're looking at identifying the function u that would give us this form, yu dot, we need to just basically identify that this is u dot. And from previous sessions, we saw that this would give us classical solution that involves an exponential of the integral of the right-hand side after dividing by u, which gives us exponential of t squared over 2. So integrating factor is just using a trick so that we simplify our left-hand side and write it in the form of the derivative of the product, yu.

So now, we identified our integration factor. It's u equals exponential of t squared over 2. Now, we can go back to our equation. And I'm going to just label it with a star here. So now, this equation is written in this form, t, and the integral of just basically derivative, which is itself. So if we use first the approach of definite integrals-- actually, we're going to switch the order, and I'm going to start with indefinite integrals first.

So using indefinite integrals, we'll be integrating both sides. And on the left-hand side, we would just be left with yu, remembering that u is just exponential of t squared over 2. On the right-hand side, we're just integrating t exponential of t squared over 2. And here, you can recognize that the derivative of exponential of t squared over 2 would have a t in front of it.

So this is actually a very simple integration. But we are in the case of differential equation where we need a constant of integration. And again, here, we would end up with two constants of integration on both sides. But given that we are dealing with a first order differential equation, we can regroup that in one constant.

And then, we can just find our solution by dividing this equation by u. And u, if you remember, is just exponential of t squared over 2, so it's equivalent to multiplying both sides by exponential of minus t squared over 2. So I'm just going to write it down. Then, it just gives us exponential minus t squared over 2 multiplied by exponential t squared over 2, which is 1 and c exponential of minus t squared over 2.

So that's our solution. But remember that we're trying to solve an initial value problem they subject to an initial condition. And our initial condition is y of 0 equals to 3, which means that here, y of 0 would give us exponential to 0, which is just a constant. So we end up with 3 equals to 1 plus c. Therefore, c is equal to 2. And the final form of the solution would just be 1 plus 2 exponential of minus t squared over 2.

So we started with the indefinite integral. So what if we would do this using definite integrals? So we don't need to start from the beginning. We just need to take the problem a few steps before when we integrated both sides of the equation. And here, specify the bounds of the integral.

So how do we want to specify the bounds of the integral? We're given an initial condition that is at t equals to 0. So that's what we want here. And we're integrating to the variable t. But one thing is important when you do that is that you have the variable t that is in the bounds of the integral. So we want the integrand to not be written in terms of the variable of integration. So it is very important to change the label of your variables in the integrand. That's how you proceed for different integrals.

So in this case then, we end up with similar-- so u dot y evaluated at t minus u dot y evaluated at 0. And from our initial conditions in the form of u, we know the value of this side of this term in the equation. And here, we're just again recognizing that this is just the derivative of exponential to s squared over 2 evaluated between 0 and t.

So here, if I just carry on the right-hand side and then go back to left-hand side in the next step, we would end up here with just t squared over 2 minus 1. Exponential of 0 is 1. And now, let's deal with this left-hand side. So again, we end up with uy minus u dot y evaluated at 0. However, u of 0 is just the function exponential to t squared over 2 evaluated at 0, which is one. And y of 0, we have it because that was our initial condition, and this is only 3.

So we have uy minus 1 dot 3, which is just basically minus 3. And on the right-hand side, we have exponential of t squared over 2 minus 1, which gives us exponential of t squared over 2 minus 1. Now bring in this minus 3 on the other side, you now have a plus 3, all of this multiplied by 1 over u, which was our exponential of minus t squared over 2. And therefore, we end up with a similar solution that we had for the different integral, 3 minus 1 is just 2. So we have minus t squared over 2 plus 1.

So using both the definite integrals approach and the indefinite integrals, we recover the same results. And the main point of this problem was really to practice using the integration factor and practice using both approaches with the definite and indefinite integrals. So this ends the problem. And I'll see you next time.