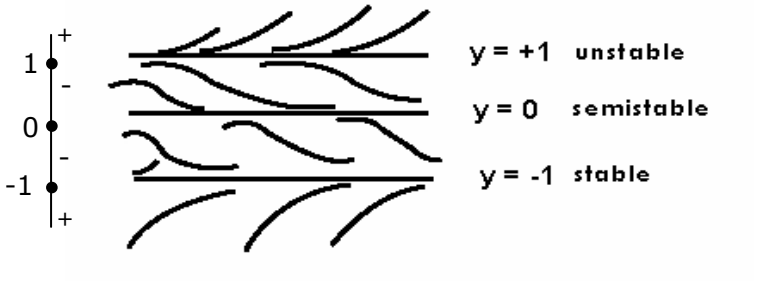
18.034, Honors Differential Equations
Prof. Jason Starr
Lecture 8

2/20/04

1. Did the autonomous ODE problem
$$y' = y^4 - y^2$$



2. Quickly talked about the "piecewise linear envelope" interpretation of existence/ uniqueness thm. Talked about difficulty w/ carrying out Picard iterates algorithm. Sequel to trying to find piecewise linear approx. of true solution.
Euler's method: $t_n = t_0 + n \bullet h$, $y_n = y(t_n)$.
$$\frac{y_{n+1} - y_n}{h} \approx y' = f(t_{n-1}, y_{n-1}).$$

Illustrated w/ $y' = y^2$, $y(0) = 2$ $h = \frac{1}{4}$, trying to find y(1). The method completely fails b/c the true solution blows up at t=1/2. Discuss applicability of numerical approximations.

3. Did better example $y' = ty(t)$, $y(0) = 2$, trying to find $y\left(\frac{1}{2}\right)$ w/ $h = 0.2^5$, $h = 0.05$. Saw that error En decreases w/ h.

4. Defined <u>order</u> of a discretization scheme. Defined 1-step and multi-step methods.

5. Gave Heun's method and explained geometrically what it does (interpolates secant slope from 2 tgt slopes).
Gave Runge-Kutta 4.

6. Someone asked about "reduction of order" to 1st-order systems, so I quickly explained this (but will return in greater detail later)