18.034, Honors Differential Equations
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Lecture 7

2/18/04

1. One more example w/ slope fields. From p. 72 Example 2.4.4. $y' = y - y^2 - 0.2\sin(t)$. Observed that on line $y = 1.2$, y′ is always negative. On line y=0.7, y′ is always positive. Thus solution curves that enter the region 0.7$0.7 \le y \le 1.2$ get trapped in this region. Then sketched the solution curves from Fig. 2.4.5. Example of using the slope field to determine "envelopes/fences".

2. Separable and exact equations. Discussed method for separable ODE's and "loss of solutions". Also discussed exact ODE's $M(t, y) + N(t, y)y = 0$. When does there exist H(t,y) s.t. $M = \frac{\partial H}{\partial t}$, $N = \frac{\partial H}{\partial y}$? M, N defined on region R.

   Prop 1: If M & N are cts. diff. on R, then H exists only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$.

   Prop 2: If R is convex, M, N are cts. diff. & $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$, then H exists.

   Did example
   $$\frac{-y}{t^2 + y^2} + \frac{t}{t^2 + y^2} y' = 0 \quad \rightsquigarrow \quad H = \tan^{-1}\left(\frac{y}{t}\right)$$, which is not single-valued, to illustrate why we need a condition on R.

3. Autonomous ODE's. $y' = f(y)$. Discussed translation invariance. Went through steps to determine equilibrium sol'ns, state line, stability of equilibrium sol'ns:

   (1) Find all zeros of f(y).
   (2) Determine the sign of f(y) b/w these values.
   (3) Draw the state line.
   (4) Sketch sol'n curve and determine stability unstability/ semi stability of equilibria.